

Chapter 11

Fluids

What is fluid?

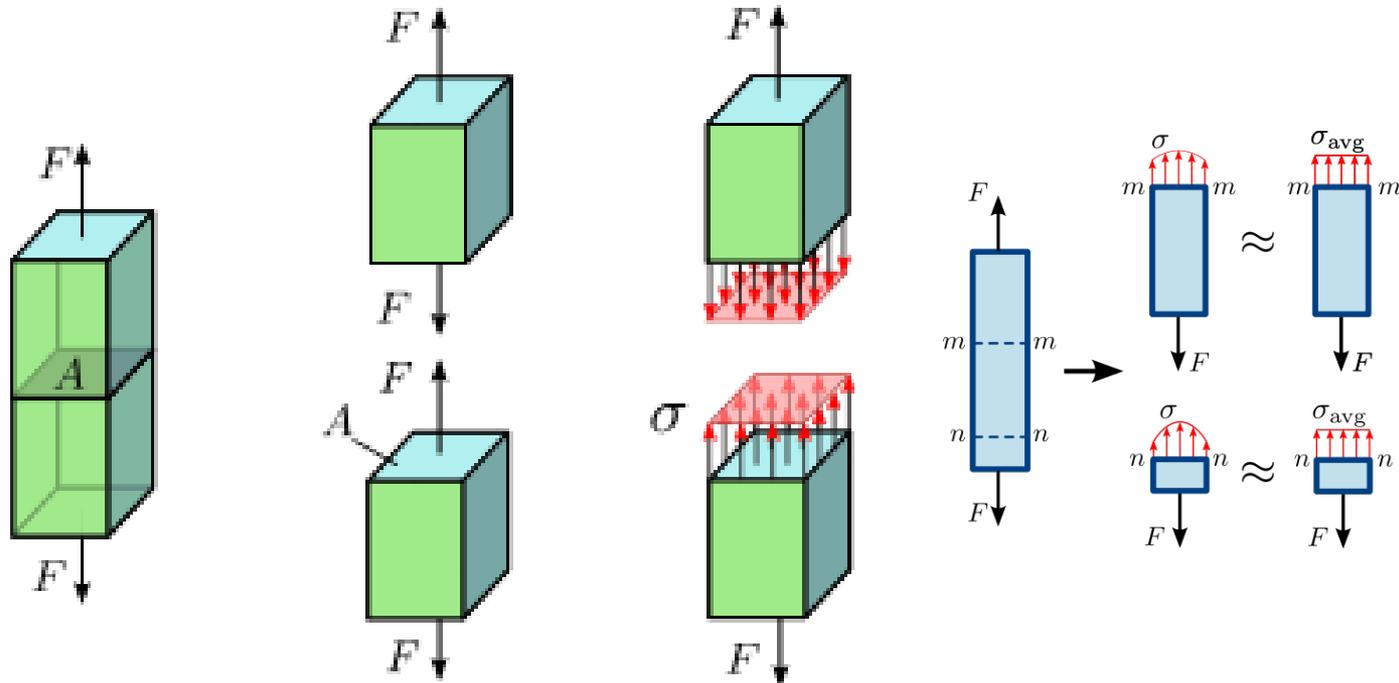
- A fluid is a collection of molecules that are randomly arranged and held together by **weak cohesive forces** and by **forces exerted by the walls of a container**.
 - Both liquids and gases are fluids.

- How do we describe **a continuum of matter**?
 - Force \rightarrow Pressure, Stress
 - Mass \rightarrow mass density

Stress (in solid)

In [continuum mechanics](#), **stress** is a [physical quantity](#) that expresses the internal [forces](#) that neighboring [particles](#) of a [continuous material](#) exert on each other. For example, when a [solid](#) vertical bar is supporting a [weight](#), each particle in the bar pulls on the particles immediately above and below it. When a [liquid](#) is under [pressure](#), each particle gets pushed inwards by all the surrounding particles, and, in [reaction](#), pushes them outwards. These macroscopic forces are actually the average of a very large number of [intermolecular forces](#) and [collisions](#) between the particles in those [molecules](#).

$$\text{Stress } \sigma = \frac{F}{A}$$



Pressure

- The **pressure** P of the fluid at the level to which the device has been submerged is the ratio of the force to the area.
- Pressure is a scalar quantity.
 - Because it is proportional to the magnitude of the force.
- If the pressure varies over an area, evaluate dF on a surface of area dA as
$$dF = P dA.$$
- Unit of pressure is **pascal** (Pa)

At any point on the surface of the object, the force exerted by the fluid is perpendicular to the surface of the object.

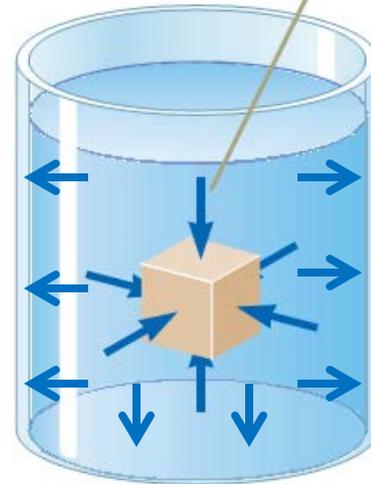


Figure 14.1 The forces exerted by a fluid on the surfaces of a submerged object.

DEFINITION OF MASS DENSITY

The mass density of a substance is the mass of a substance divided by its volume:

$$\rho = \frac{m}{V}$$

SI Unit of Mass Density: kg/m³

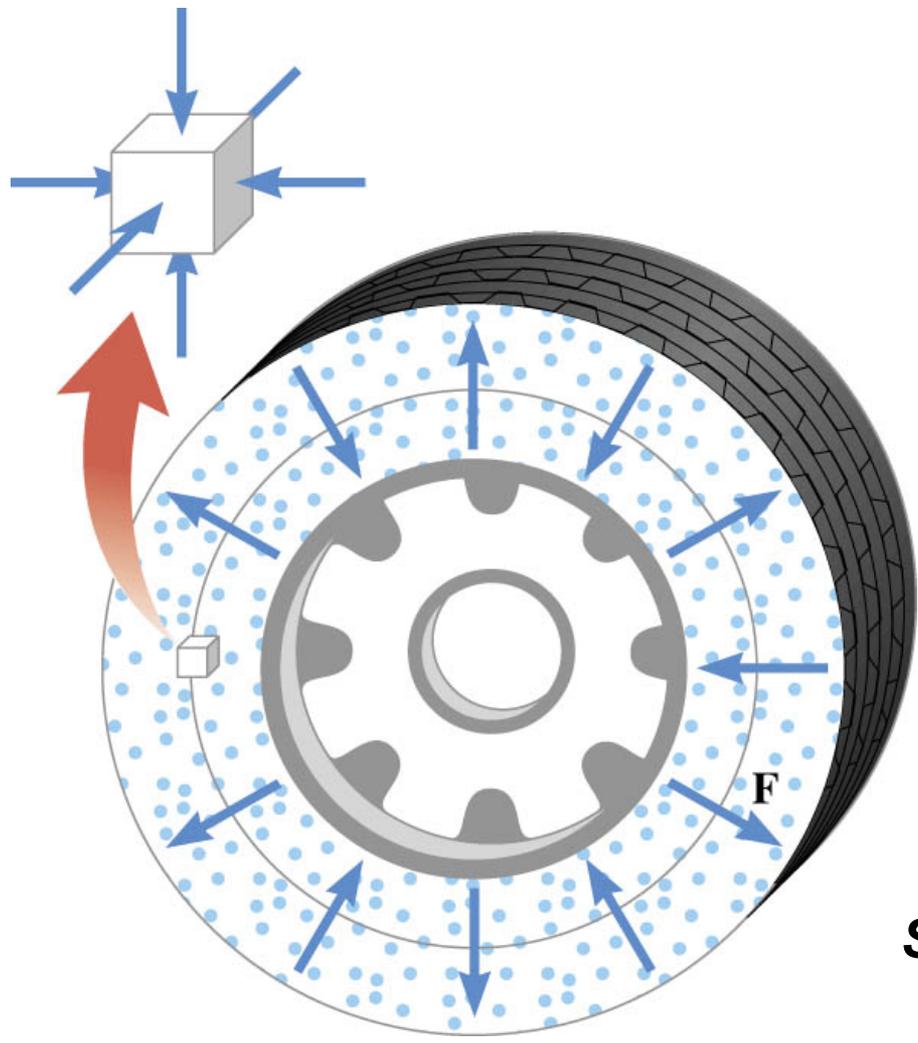
11.1 Mass Density

Table 11.1 Mass Densities^a
of Common Substances

Substance	Mass Density ρ (kg/m ³)
Solids	
Aluminum	2700
Brass	8470
Concrete	2200
Copper	8890
Diamond	3520
Gold	19 300
Ice	917
Iron (steel)	7860
Lead	11 300
Quartz	2660
Silver	10 500
Wood (yellow pine)	550
Liquids	
Blood (whole, 37 °C)	1060
Ethyl alcohol	806
Mercury	13 600
Oil (hydraulic)	800
Water (4 °C)	1.000×10^3
Gases	
Air	1.29
Carbon dioxide	1.98
Helium	0.179
Hydrogen	0.0899
Nitrogen	1.25
Oxygen	1.43

^a Unless otherwise noted, densities are given at 0 °C and 1 atm pressure.

11.2 Pressure



$$P = \frac{F}{A}$$

SI Unit of Pressure: 1 N/m² = 1Pa

Pascal

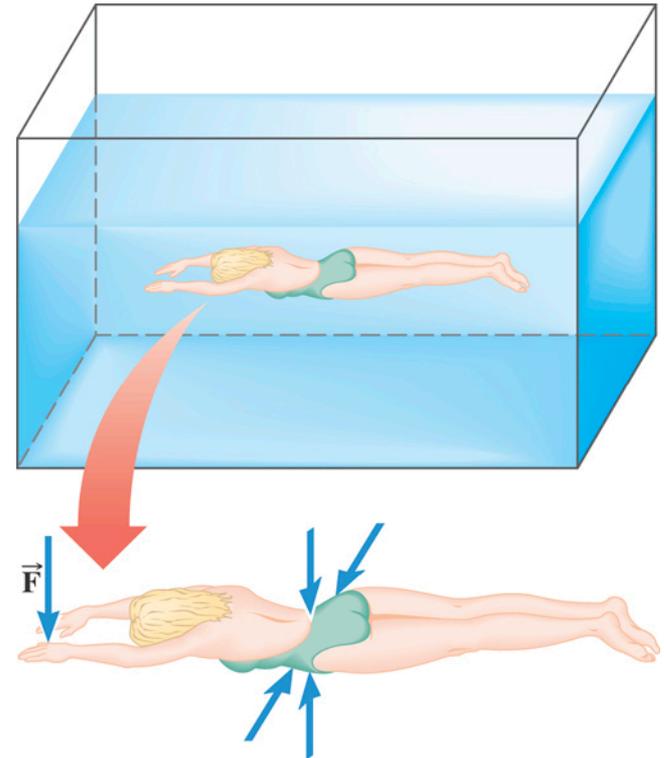


11.2 Pressure

Example 2 The Force on a Swimmer

Suppose the pressure acting on the back of a swimmer's hand is $1.2 \times 10^5 \text{ Pa}$. The surface area of the back of the hand is $8.4 \times 10^{-3} \text{ m}^2$.

- (a) Determine the magnitude of the force that acts on it.
- (b) Discuss the direction of the force.

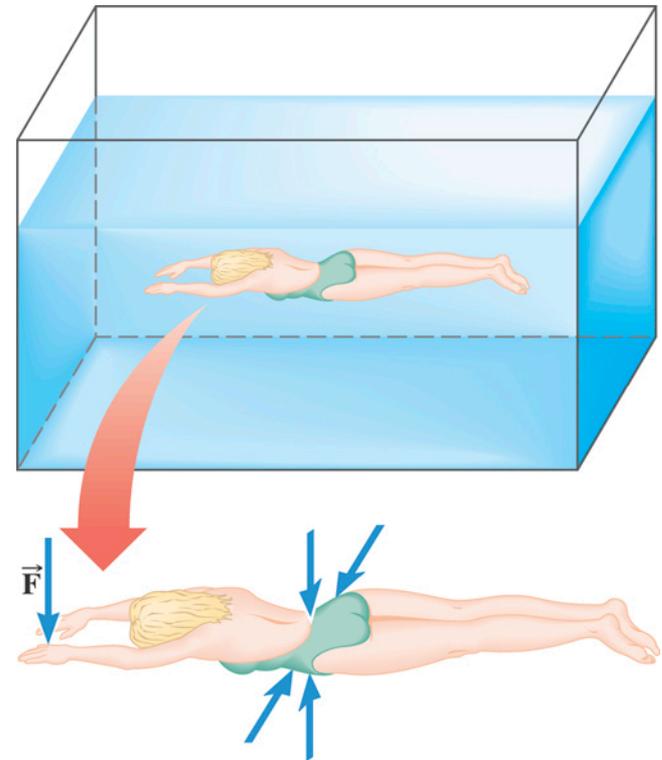


11.2 Pressure

$$P = \frac{F}{A}$$

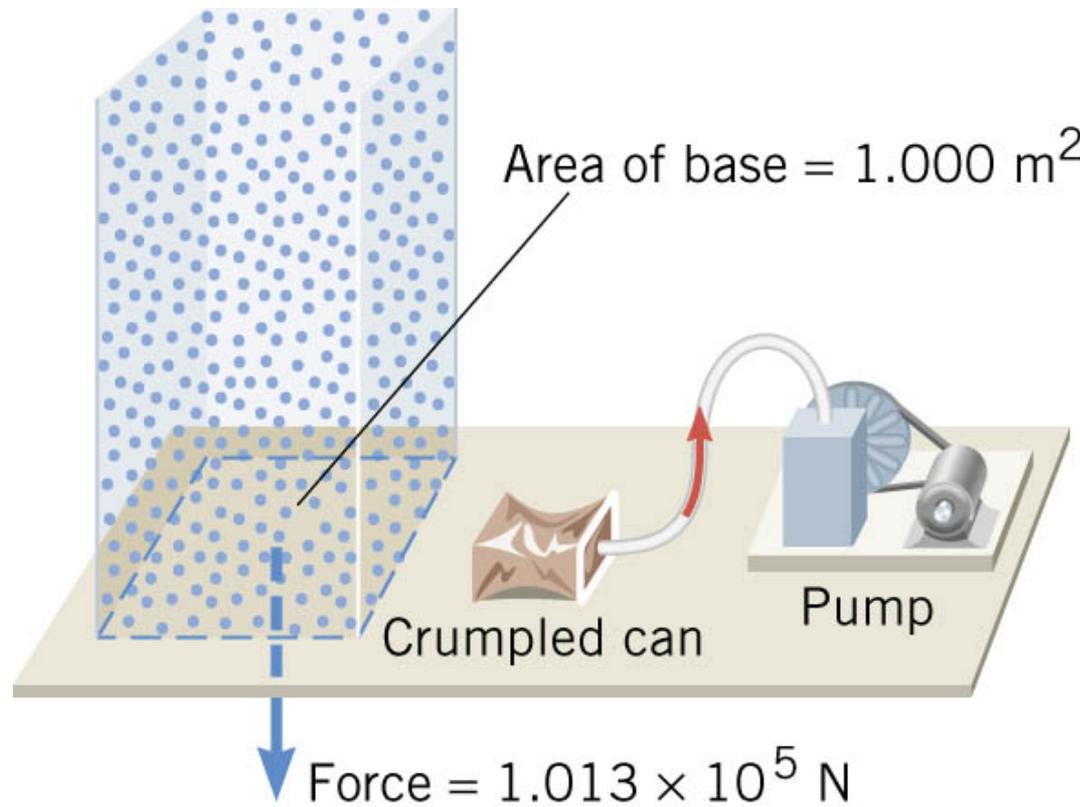
$$F = PA = (1.2 \times 10^5 \text{ N/m}^2)(8.4 \times 10^{-3} \text{ m}^2) \\ = 1.0 \times 10^3 \text{ N}$$

Since the water pushes perpendicularly against the back of the hand, the force is directed downward in the drawing.

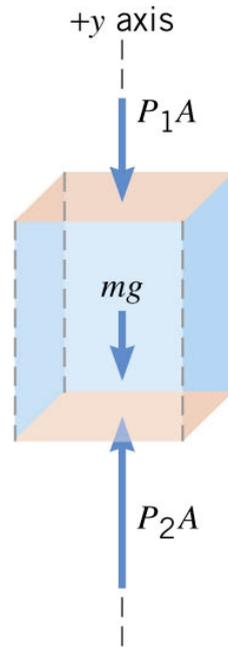
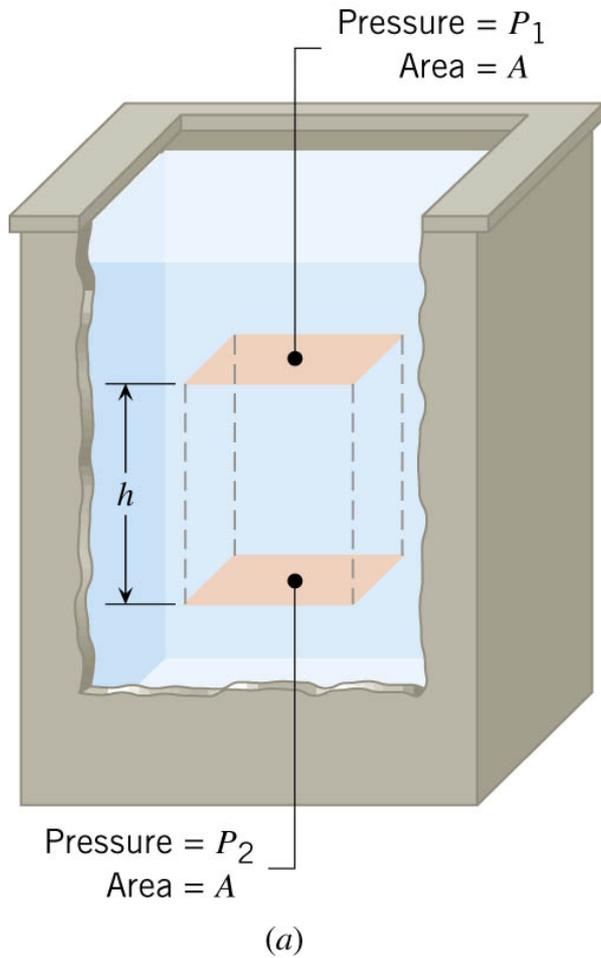


11.2 Pressure

Atmospheric Pressure at Sea Level: $1.013 \times 10^5 \text{ Pa} = 1 \text{ atmosphere}$



11.3 Pressure and Depth in a Static Fluid



(b) Free-body diagram of the column

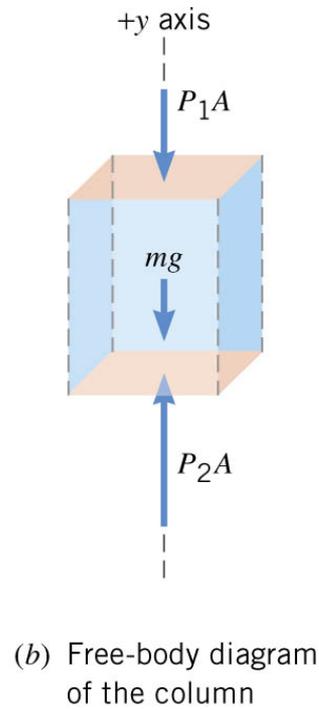
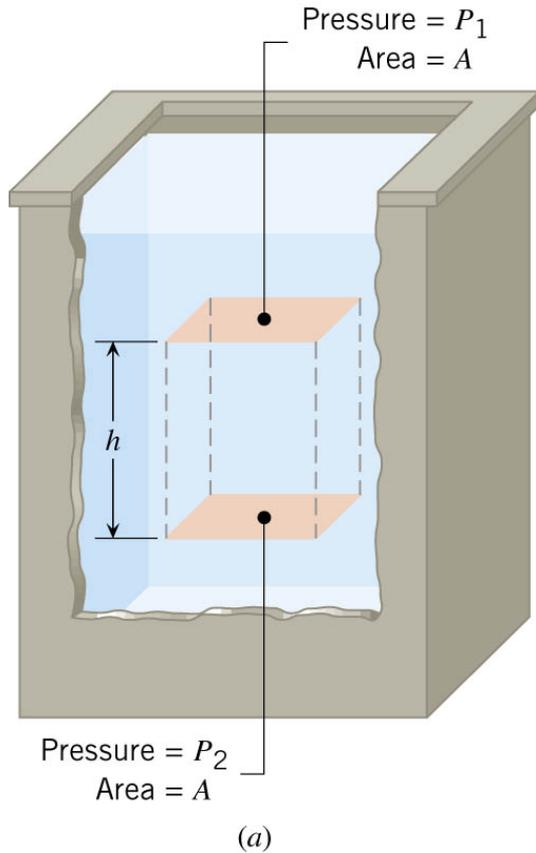
$$\sum F_y = P_2A - P_1A - mg = 0$$



$$P_2A = P_1A + mg$$

$$m = V\rho$$

11.3 Pressure and Depth in a Static Fluid



$$P_2A = P_1A + \rho Vg$$

$V = Ah$

$$P_2A = P_1A + \rho Ahg$$

$$P_2 = P_1 + \rho hg$$

Conceptual Example 3 The Hoover Dam

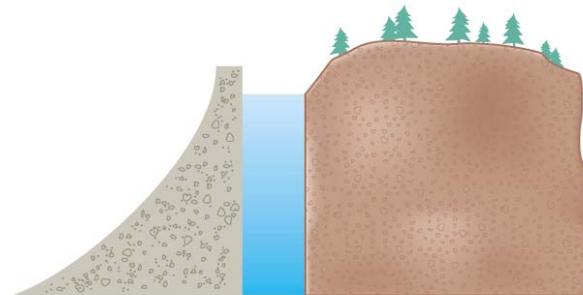
Lake Mead is the largest wholly artificial reservoir in the United States. The water in the reservoir backs up behind the dam for a considerable distance (120 miles).

Suppose that all the water in Lake Mead were removed except a relatively narrow vertical column.

Would the Hoover Dam still be needed to contain the water, or could a much less massive structure do the job?



(a)

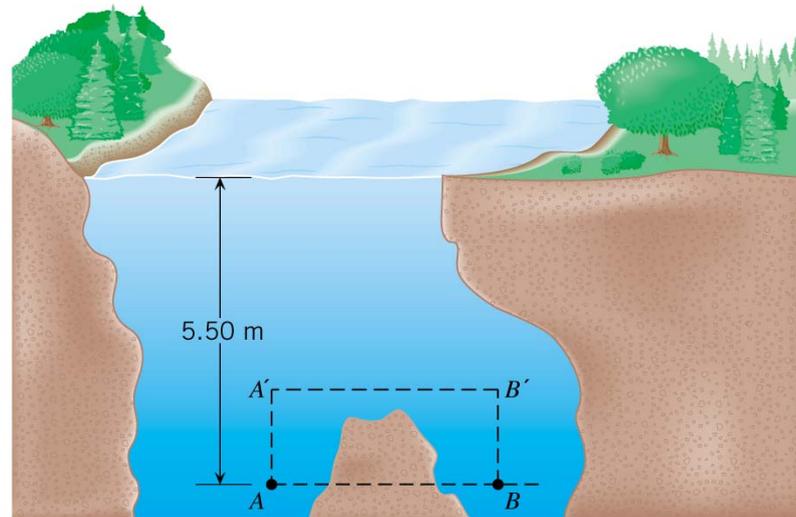


(b)

11.3 Pressure and Depth in a Static Fluid

Example 4 The Swimming Hole

Points A and B are located a distance of 5.50 m beneath the surface of the water. Find the pressure at each of these two locations.

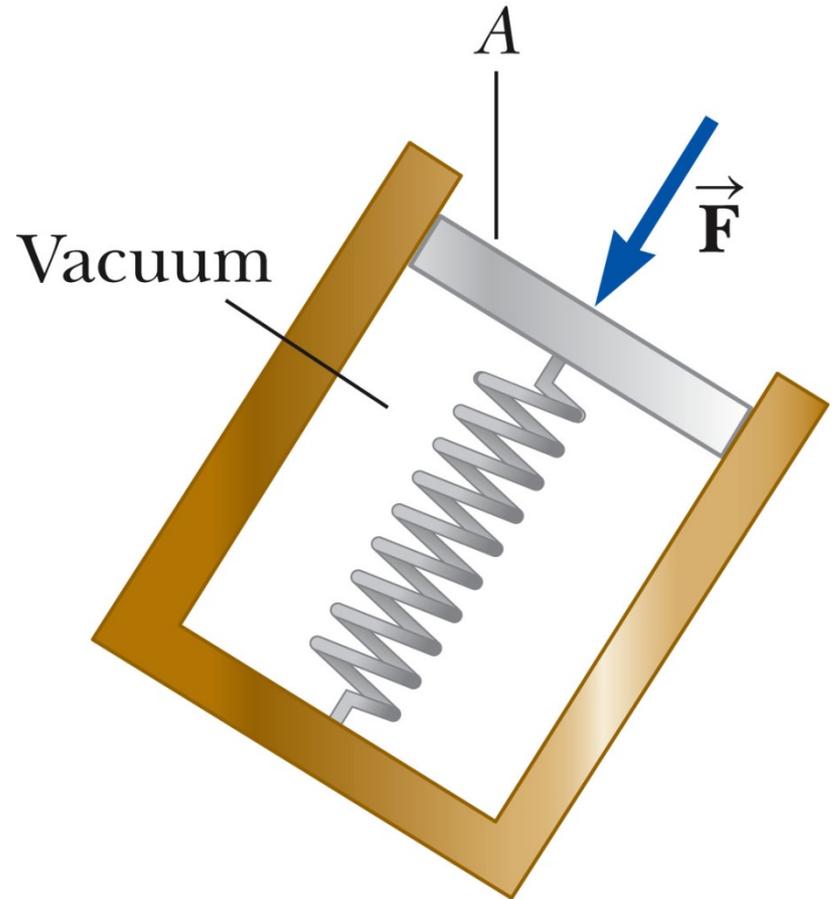


$$P_2 = P_1 + \rho gh$$

$$P_2 = \overbrace{(1.01 \times 10^5 \text{ Pa})}^{\text{atmospheric pressure}} + (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.50 \text{ m})$$
$$= 1.55 \times 10^5 \text{ Pa}$$

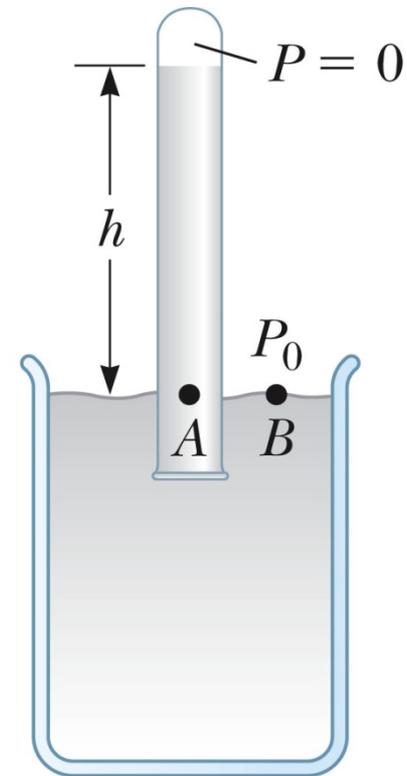
Measuring Pressure

- The spring is calibrated by a known force.
- The force due to the fluid presses on the top of the piston and compresses the spring.
- The force the fluid exerts on the piston is then measured.



Pressure Measurements: Barometer

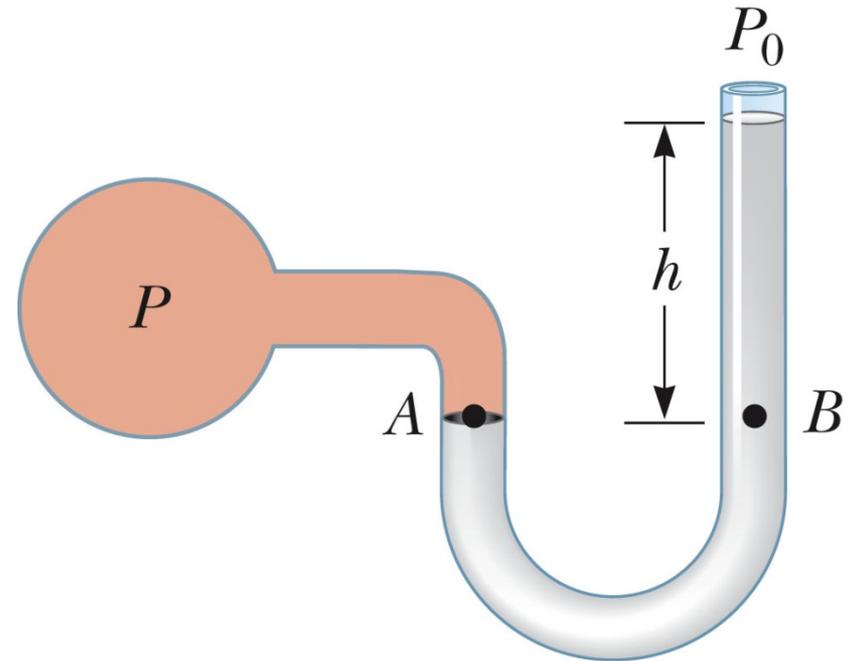
- Invented by Torricelli
- A long closed tube is filled with mercury and inverted in a dish of mercury.
 - The closed end is nearly a vacuum.
- Measures atmospheric pressure as $P_o = \rho_{\text{Hg}} g h$
- 1 atm = 0.760 m (of Hg)



a

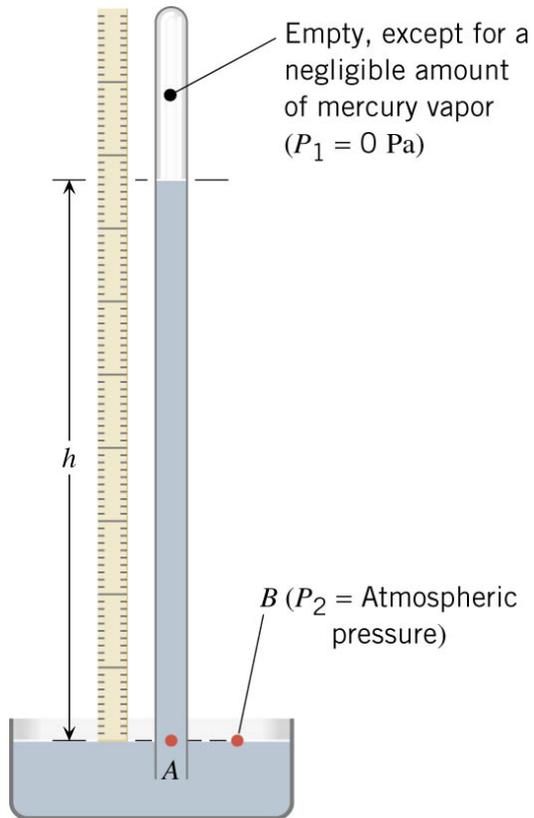
Pressure Measurements: Manometer

- A device for measuring the pressure of a gas contained in a vessel.
- One end of the U-shaped tube is open to the atmosphere.
- The other end is connected to the pressure to be measured.
- Pressure at B is $P = P_0 + \rho gh$
- The height can be calibrated to measure the pressure.

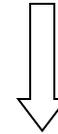


b

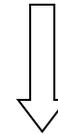
11.4 Pressure Gauges



$$P_2 = P_1 + \rho gh$$



$$P_{atm} = \rho gh$$



$$h = \frac{P_{atm}}{\rho g} = \frac{(1.01 \times 10^5 \text{ Pa})}{(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)}$$

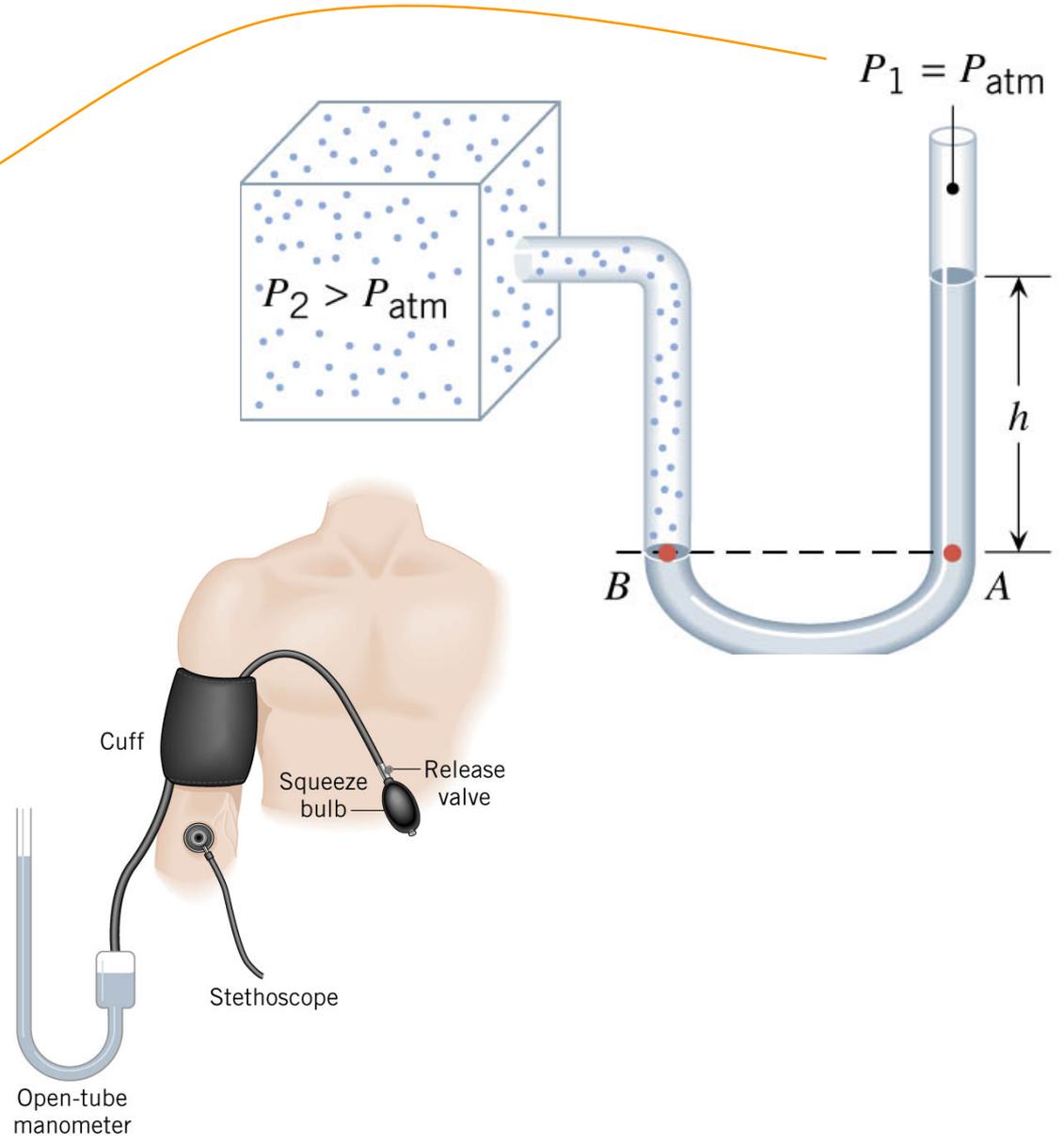
$$= 0.760 \text{ m} = 760 \text{ mm}$$

11.4 Pressure Gauges

$$P_2 = P_B = P_A$$
$$P_A = P_1 + \rho gh$$

absolute pressure

$$\underbrace{P_2 - P_{atm}}_{\text{gauge pressure}} = \rho gh$$



11.5 Pascal's Principle

PASCAL'S PRINCIPLE

Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and enclosing walls.

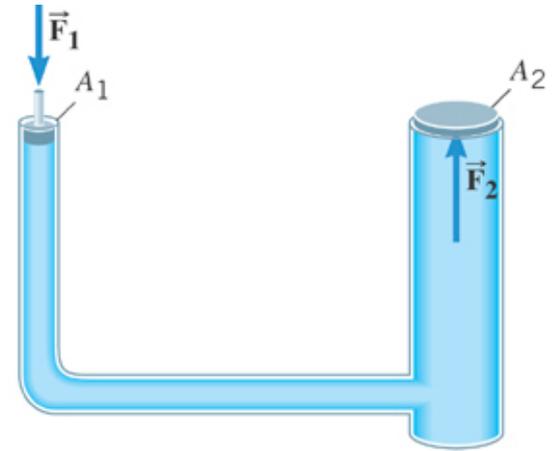
Applications:

Hydraulic brakes

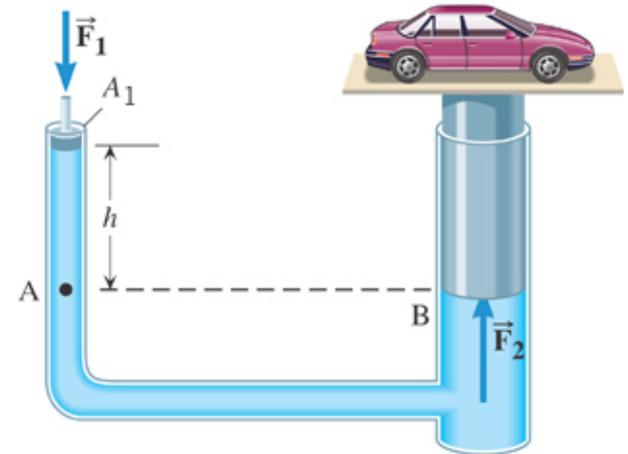
Car lifts

Hydraulic jacks

Forklifts

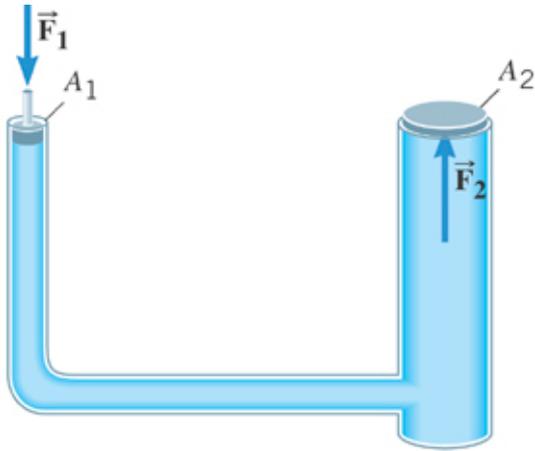


(a)



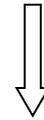
(b)

11.5 Pascal's Principle



(a)

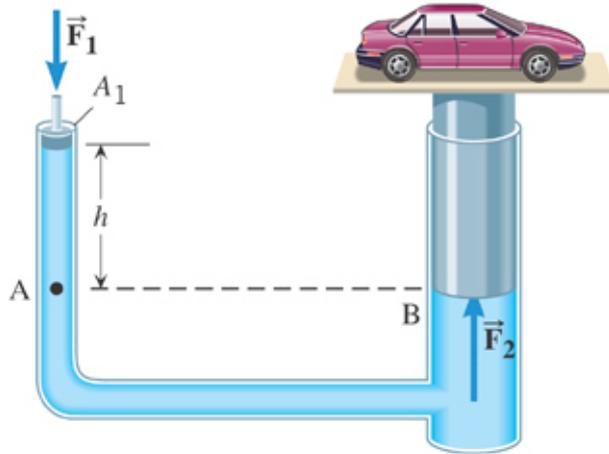
$$P_2 = P_1 + \rho g(0 \text{ m})$$



$$\frac{F_2}{A_2} = \frac{F_1}{A_1}$$



$$F_2 = F_1 \left(\frac{A_2}{A_1} \right)$$



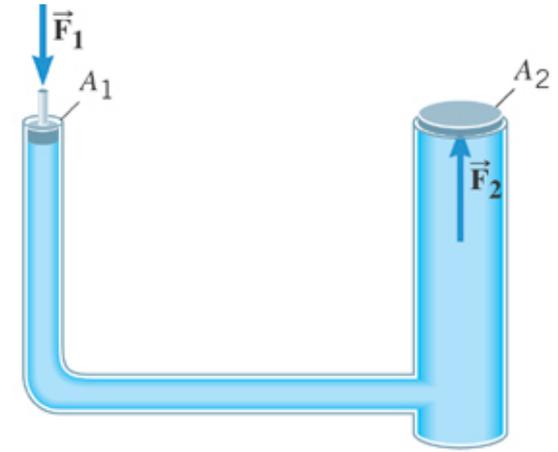
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11.5 Pascal's Principle

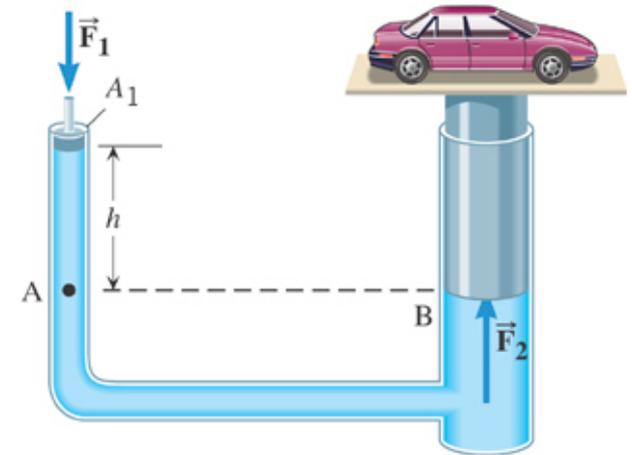
Example 7 A Car Lift

The input piston has a radius of 0.0120 m and the output plunger has a radius of 0.150 m.

The combined weight of the car and the plunger is 20500 N. Suppose that the input piston has a negligible weight and the bottom surfaces of the piston and plunger are at the same level. What is the required input force?



(a)

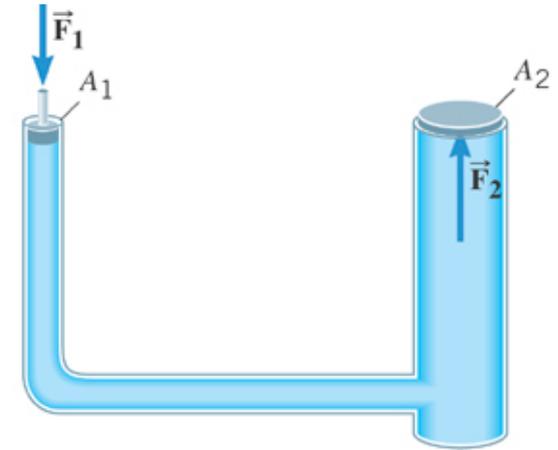


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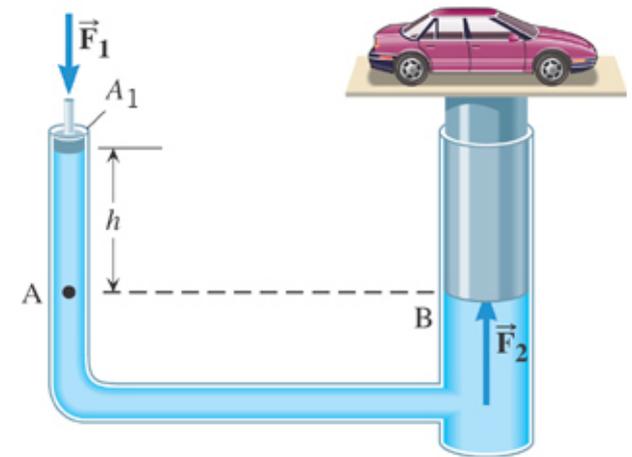
11.5 Pascal's Principle

$$F_2 = F_1 \left(\frac{A_2}{A_1} \right)$$

$$F_2 = (20500 \text{ N}) \frac{\pi(0.0120 \text{ m})^2}{\pi(0.150 \text{ m})^2} = 131 \text{ N}$$

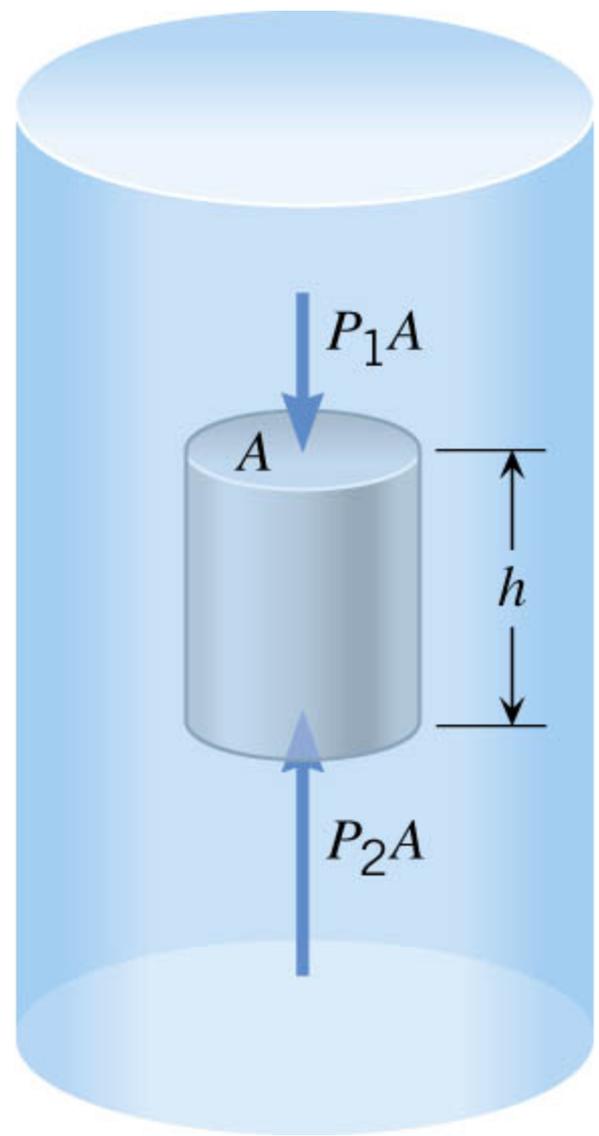


(a)



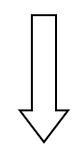
(b)

11.6 Archimedes' Principle



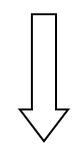
$$P_2 - P_1 = \rho gh$$

$$F_B = P_2A - P_1A = (P_2 - P_1)A$$



$$V = hA$$

$$F_B = \rho ghA$$



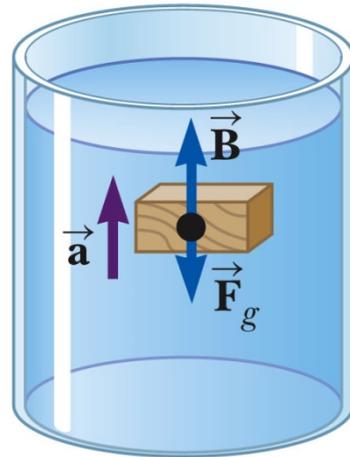
$$F_B = \underbrace{\rho V}_{\text{mass of displaced fluid}} g$$

ARCHIMEDES' PRINCIPLE

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; **the magnitude of the buoyant force equals the weight of the fluid that the object displaces (replaces):**

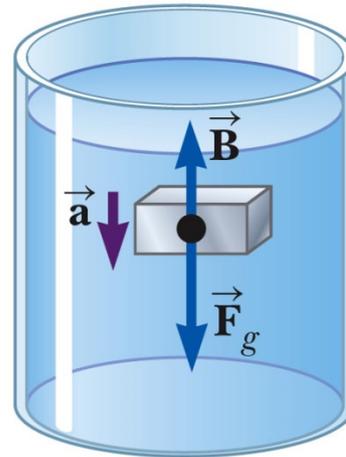
$$\underbrace{F_B}_{\text{Magnitude of buoyant force}} = \underbrace{W_{\text{fluid}}}_{\text{Weight of displaced fluid}}$$

$$\rho_{\text{obj}} < \rho_{\text{fluid}}$$



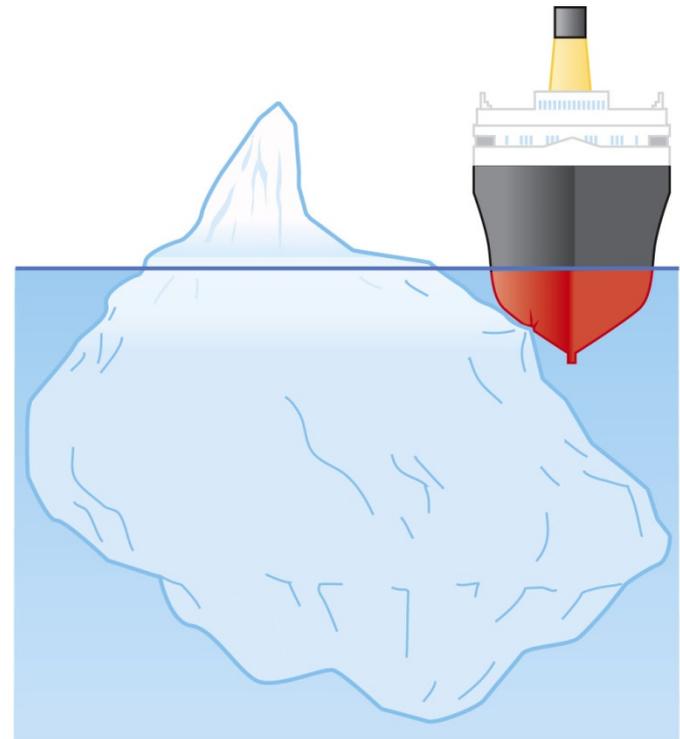
a

$$\rho_{\text{obj}} > \rho_{\text{fluid}}$$



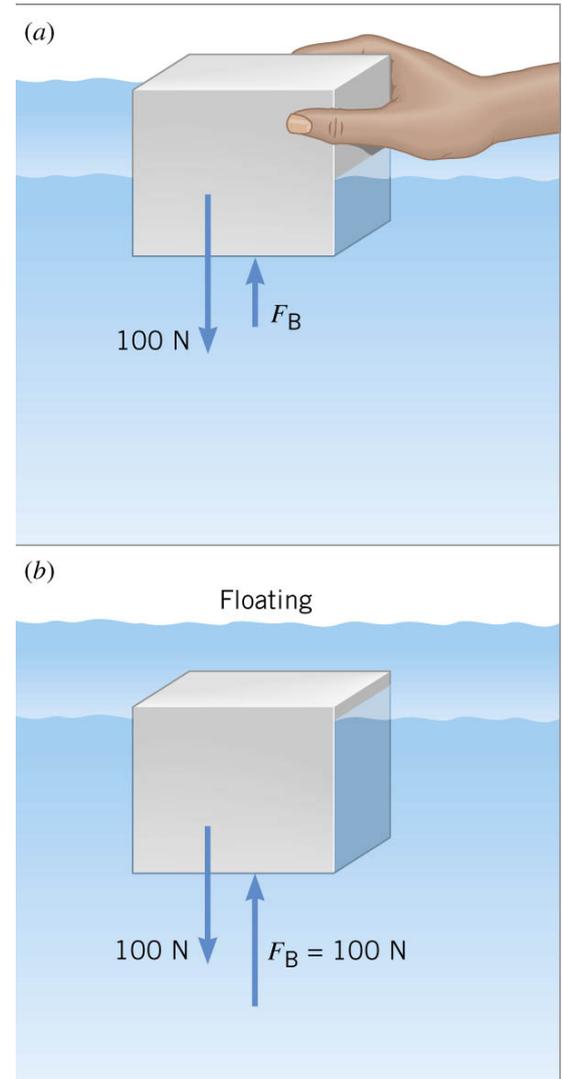
b

- What fraction of the iceberg is below water?
- The iceberg is only partially submerged and so $V_{\text{disp}} / V_{\text{ice}} = \rho_{\text{ice}} / \rho_{\text{seawater}}$ applies
- About 89% of the ice is below the water's surface.



11.6 Archimedes' Principle

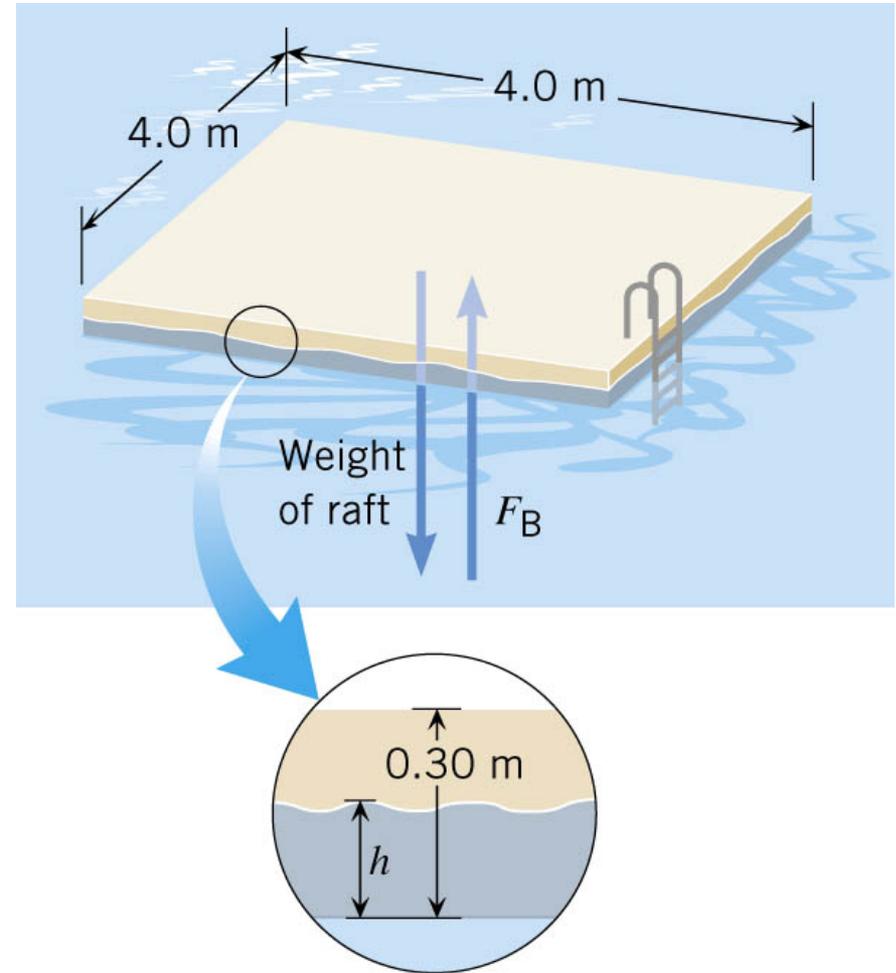
If the object is floating then the magnitude of the buoyant force is equal to the magnitude of its weight.



11.6 Archimedes' Principle

Example 9 A Swimming Raft

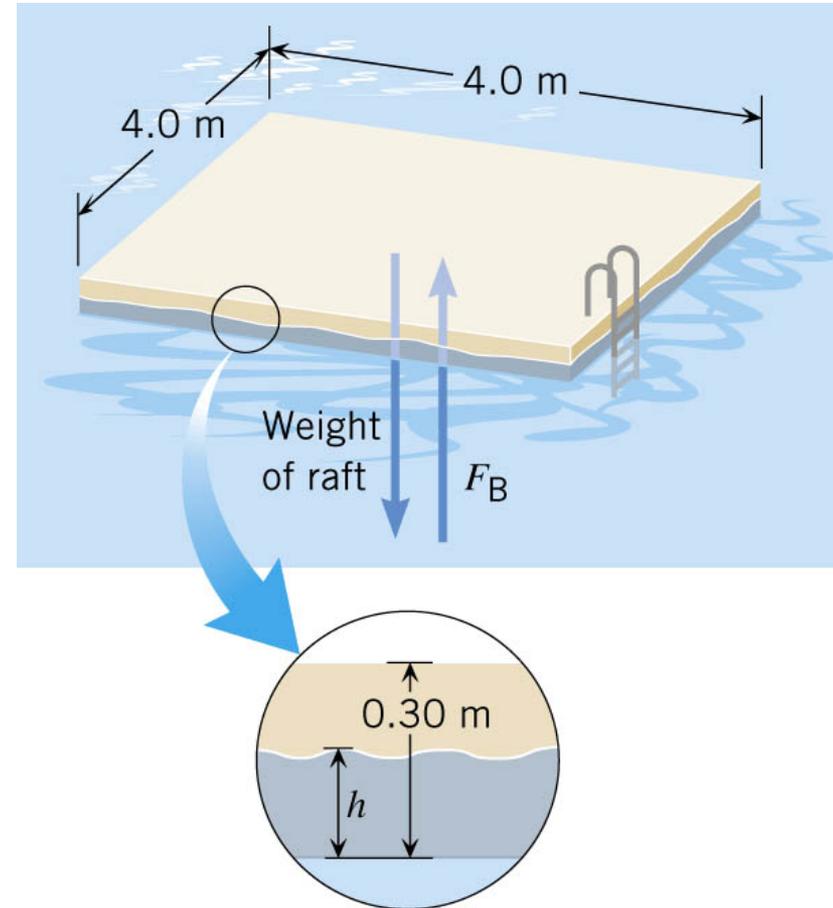
The raft is made of solid square pinewood. Determine whether the raft floats in water and if so, how much of the raft is beneath the surface.



11.6 Archimedes' Principle

$$V_{\text{raft}} = (4.0 \text{ m})(4.0 \text{ m})(0.30 \text{ m}) = 4.8 \text{ m}^3$$

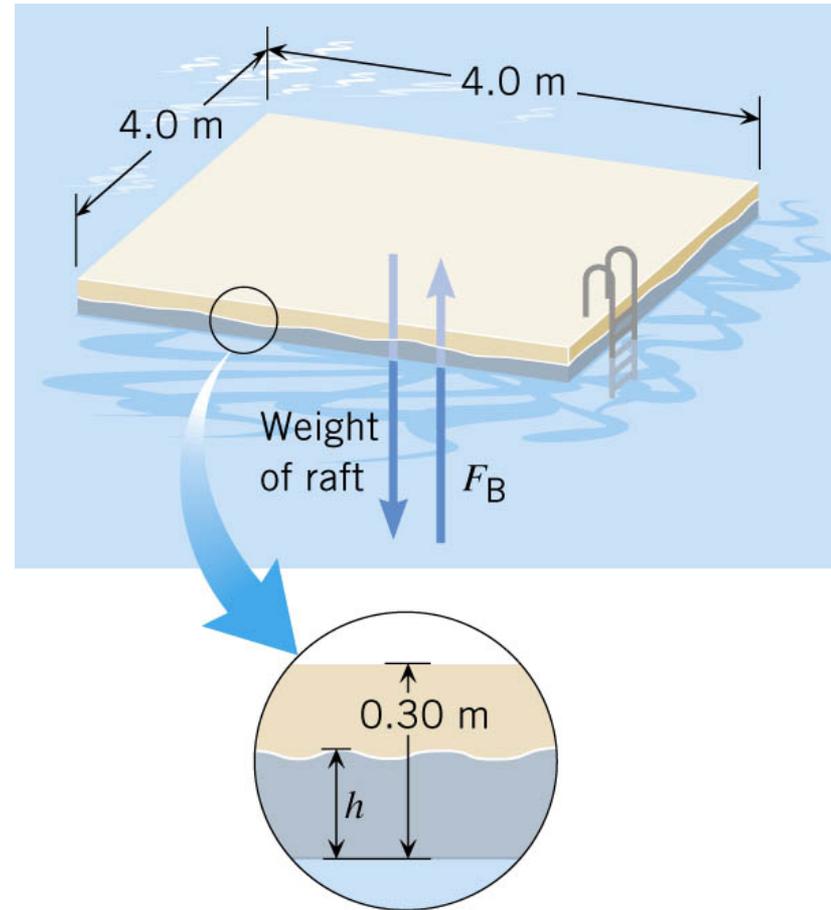
$$\begin{aligned} F_B^{\text{max}} &= \rho V g = \rho_{\text{water}} V_{\text{water}} g \\ &= (1000 \text{ kg/m}^3)(4.8 \text{ m}^3)(9.80 \text{ m/s}^2) \\ &= 47000 \text{ N} \end{aligned}$$



11.6 Archimedes' Principle

$$\begin{aligned}W_{raft} &= m_{raft} g = \rho_{pine} V_{raft} g \\&= (550 \text{ kg/m}^3)(4.8 \text{ m}^3)(9.80 \text{ m/s}^2) \\&= 26000 \text{ N} < 47000 \text{ N}\end{aligned}$$

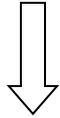
The raft floats!



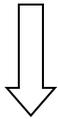
11.6 Archimedes' Principle

If the raft is floating:

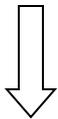
$$W_{\text{raft}} = F_B$$



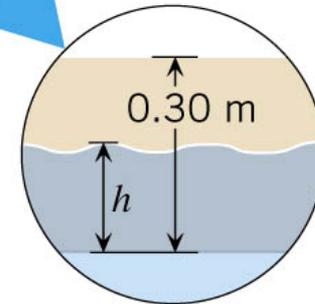
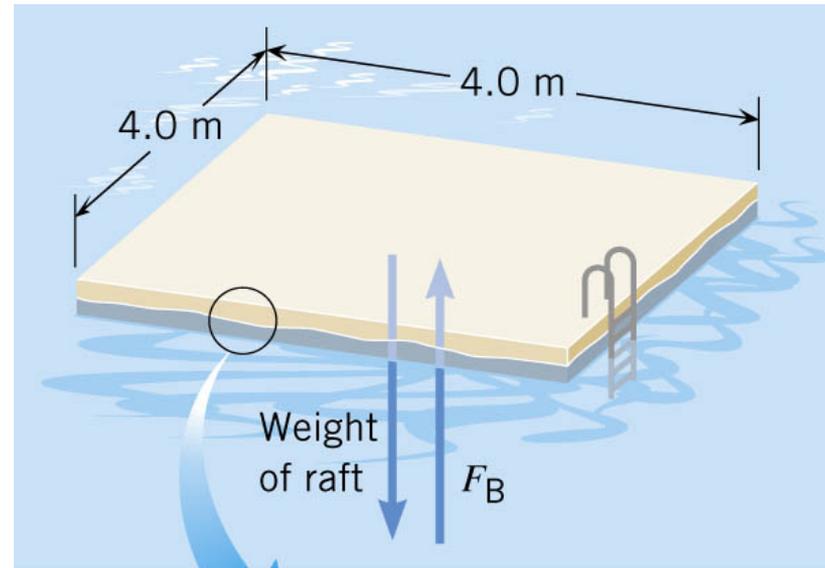
$$26000 \text{ N} = \rho_{\text{water}} V_{\text{water}} g$$



$$26000 \text{ N} = (1000 \text{ kg/m}^3)(4.0 \text{ m})(4.0 \text{ m})h(9.80 \text{ m/s}^2)$$



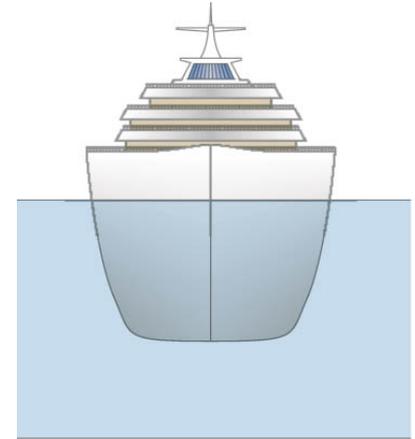
$$h = \frac{26000 \text{ N}}{(1000 \text{ kg/m}^3)(4.0 \text{ m})(4.0 \text{ m})(9.80 \text{ m/s}^2)} = 0.17 \text{ m}$$



11.6 Archimedes' Principle

Conceptual Example 10 How Much Water is Needed to Float a Ship?

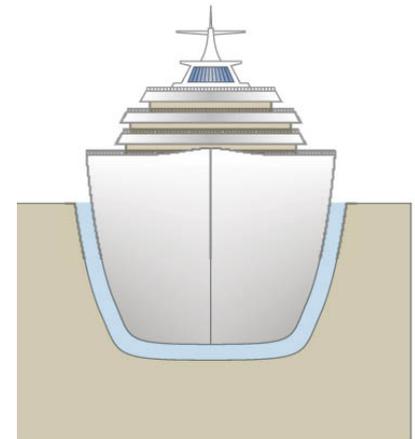
A ship floating in the ocean is a familiar sight. But is all that water really necessary? Can an ocean vessel float in the amount of water than a swimming pool contains?



(a)



(b)

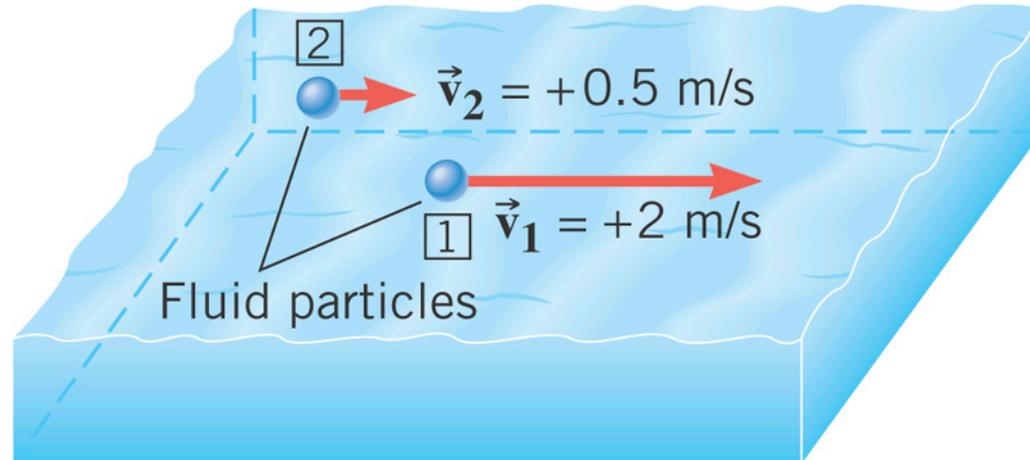


(c)

11.7 Fluids in Motion

In **steady flow** the velocity of the fluid particles at any point is constant as time passes.

Unsteady flow exists whenever the velocity of the fluid particles at a point changes as time passes.



Turbulent flow is an extreme kind of unsteady flow in which the velocity of the fluid particles at a point change erratically in both magnitude and direction.

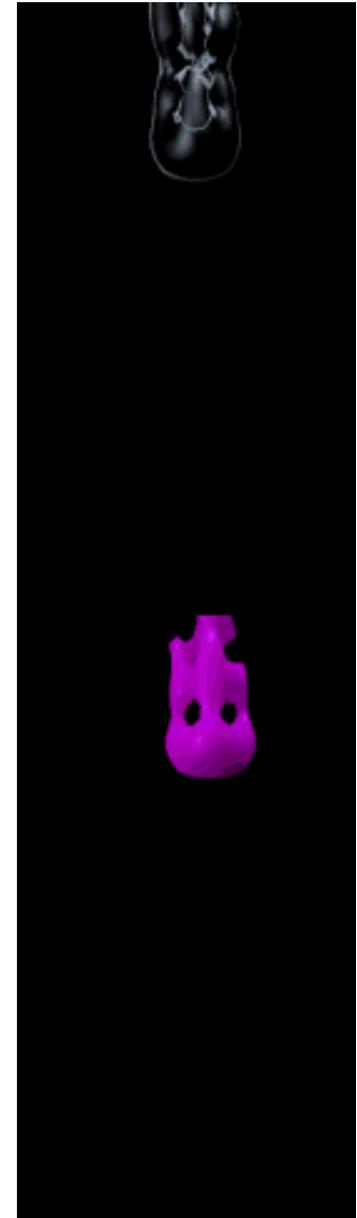
11.7 Fluids in Motion

Fluid flow can be **compressible** or **incompressible**.
Most liquids are
nearly incompressible. (ρ change in space/time?)

Fluid flow can be **viscous** or **nonviscous**. (surface
tension, friction? (between molecules flowing with
different velocities)

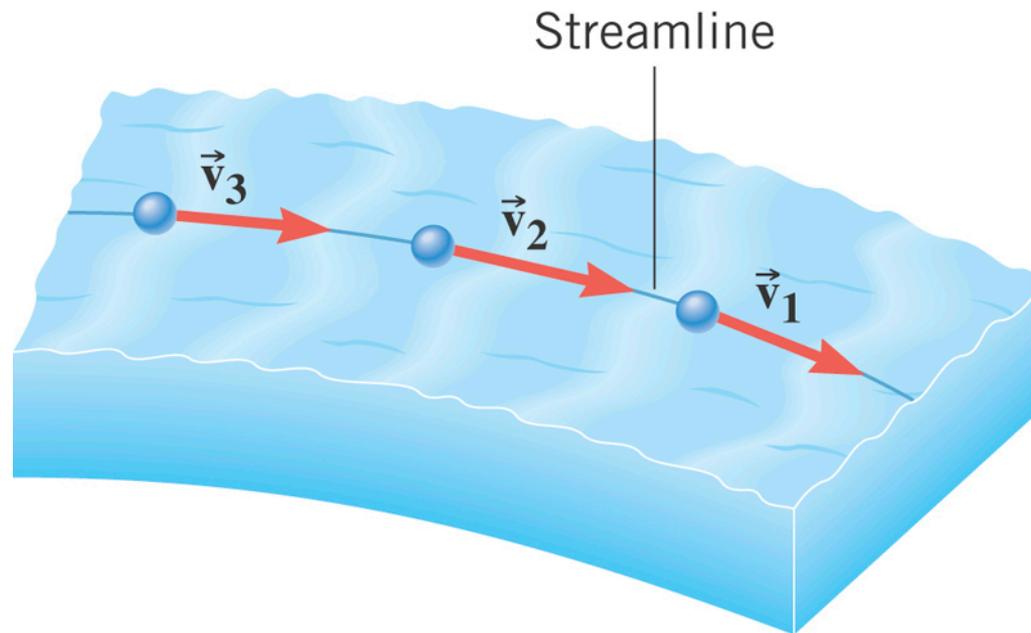
An incompressible, nonviscous fluid is called an **ideal
fluid**.

Zero viscosity is observed only at very low
temperatures such as in superfluids (He).



11.7 Fluids in Motion

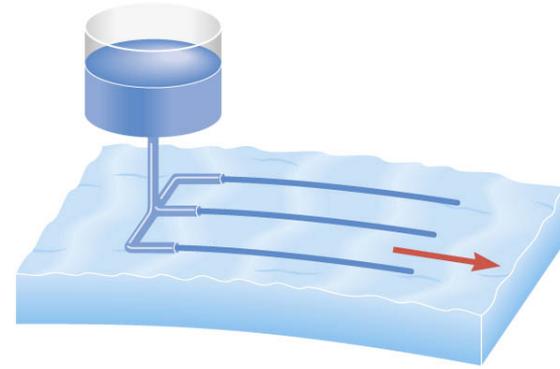
When the flow is steady, **streamlines** are often used to represent the trajectories of the fluid particles.



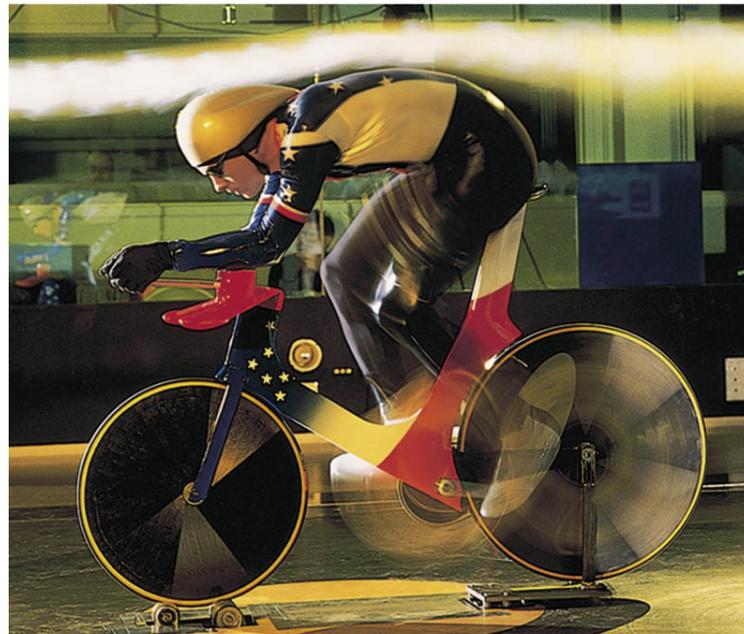
Field quantity $\mathbf{v}(\mathbf{r},t)$

11.7 Fluids in Motion

Making streamlines with dye and smoke.



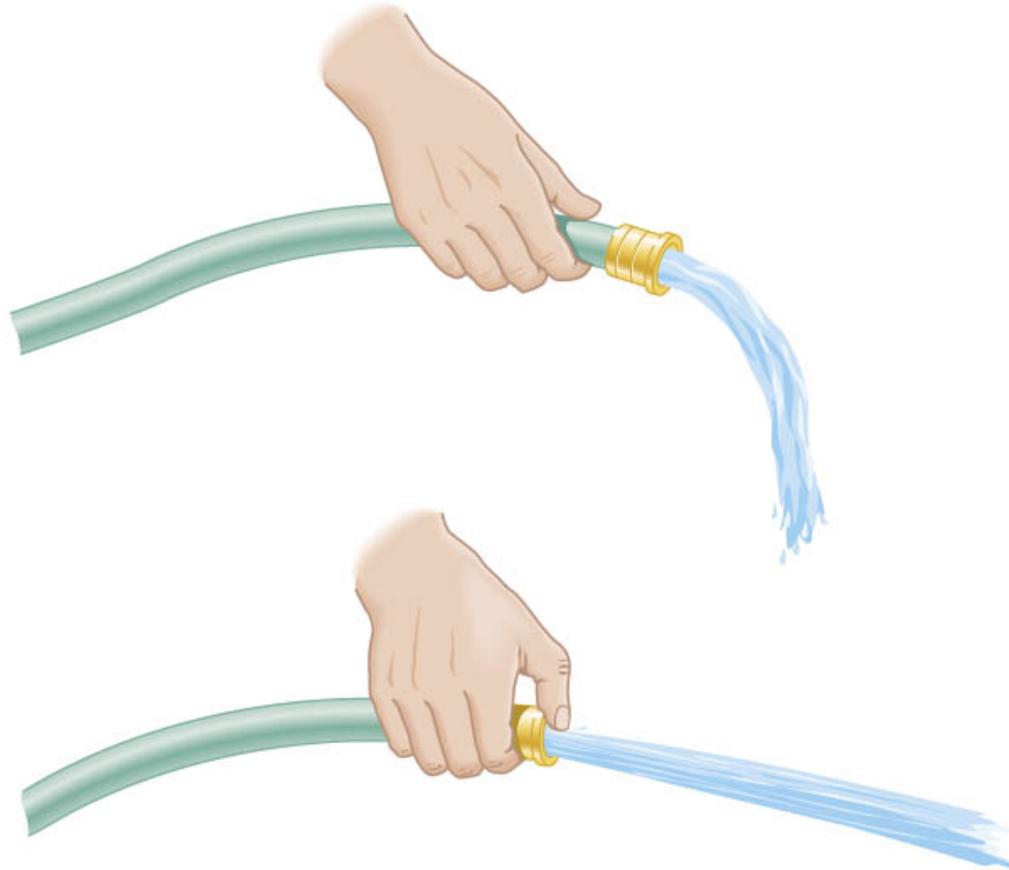
(a)



(b)

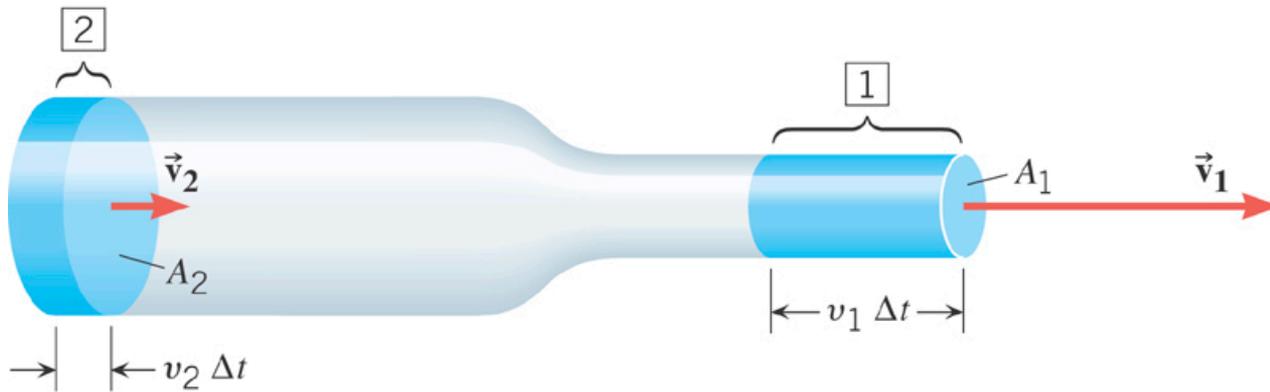
11.8 The Equation of Continuity

The mass of fluid per second that flows through a tube is called the ***mass flow rate***.



11.8 The Equation of Continuity

$$\Delta m = \rho V = \rho A \underbrace{v \Delta t}_{\text{distance}}$$



$$\frac{\Delta m_2}{\Delta t} = \rho_2 A_2 v_2$$

$$\frac{\Delta m_1}{\Delta t} = \rho_1 A_1 v_1$$

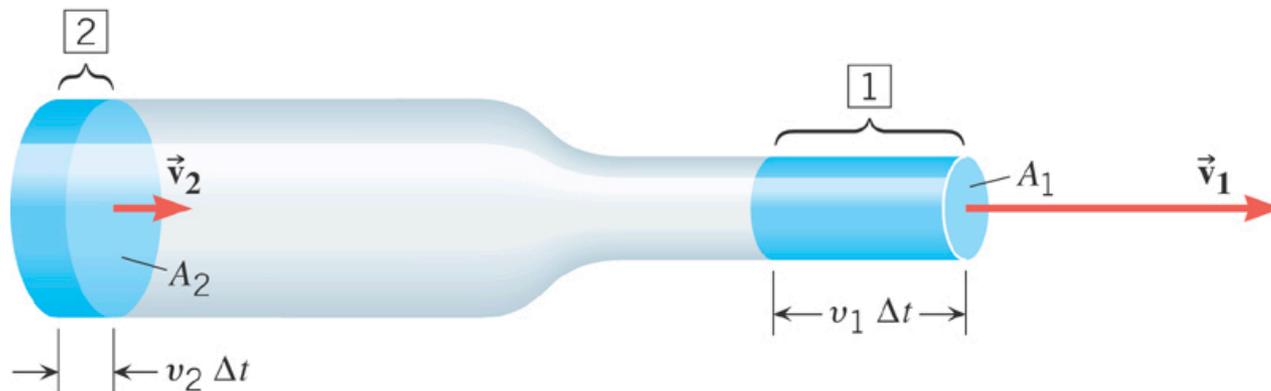
11.8 The Equation of Continuity

EQUATION OF CONTINUITY

The mass flow rate has the same value at every position along a tube that has a single entry and a single exit for fluid flow.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

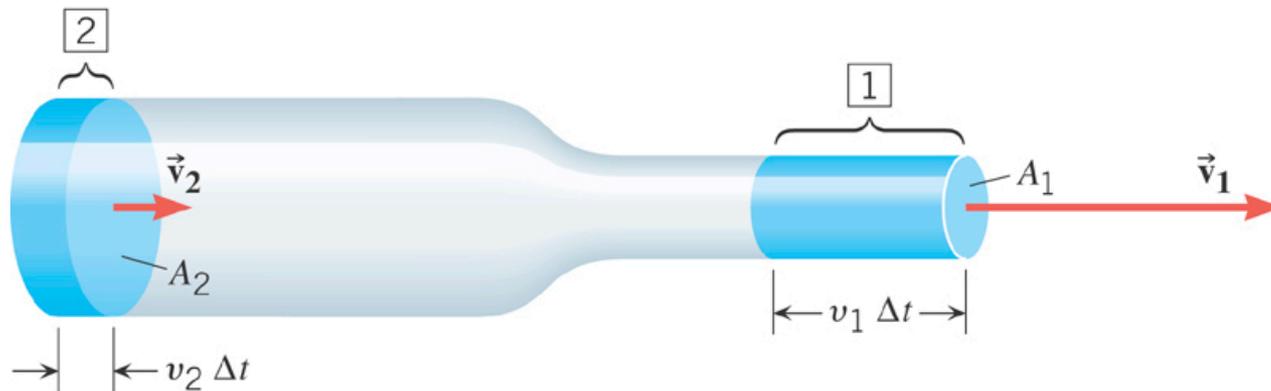
SI Unit of Mass Flow Rate: kg/s



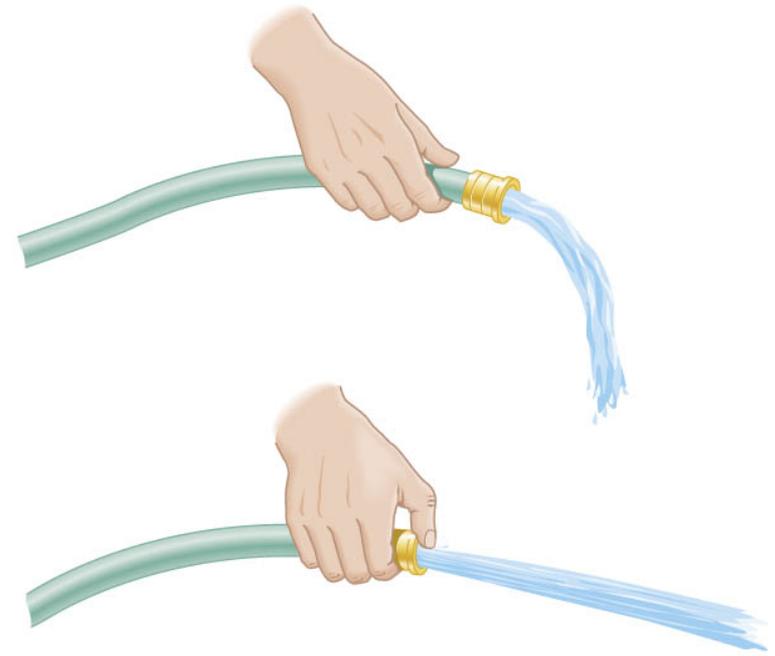
11.8 The Equation of Continuity

Incompressible fluid: $A_1 v_1 = A_2 v_2$

Volume flow rate Q : $Q = Av$



11.8 The Equation of Continuity



Example 12 A Garden Hose

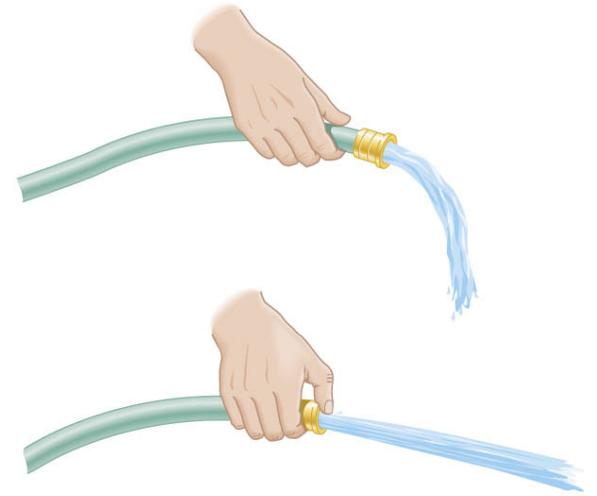
A garden hose has an unobstructed opening with a cross sectional area of $2.85 \times 10^{-4} \text{m}^2$. It fills a bucket with a volume of $8.00 \times 10^{-3} \text{m}^3$ in 30 seconds.

Find the speed of the water that leaves the hose through (a) the unobstructed opening and (b) an obstructed opening with half as much area.

11.8 The Equation of Continuity

(a) $Q = Av$

$$v = \frac{Q}{A} = \frac{(8.00 \times 10^{-3} \text{ m}^3) / (30.0 \text{ s})}{2.85 \times 10^{-4} \text{ m}^2} = 0.936 \text{ m/s}$$



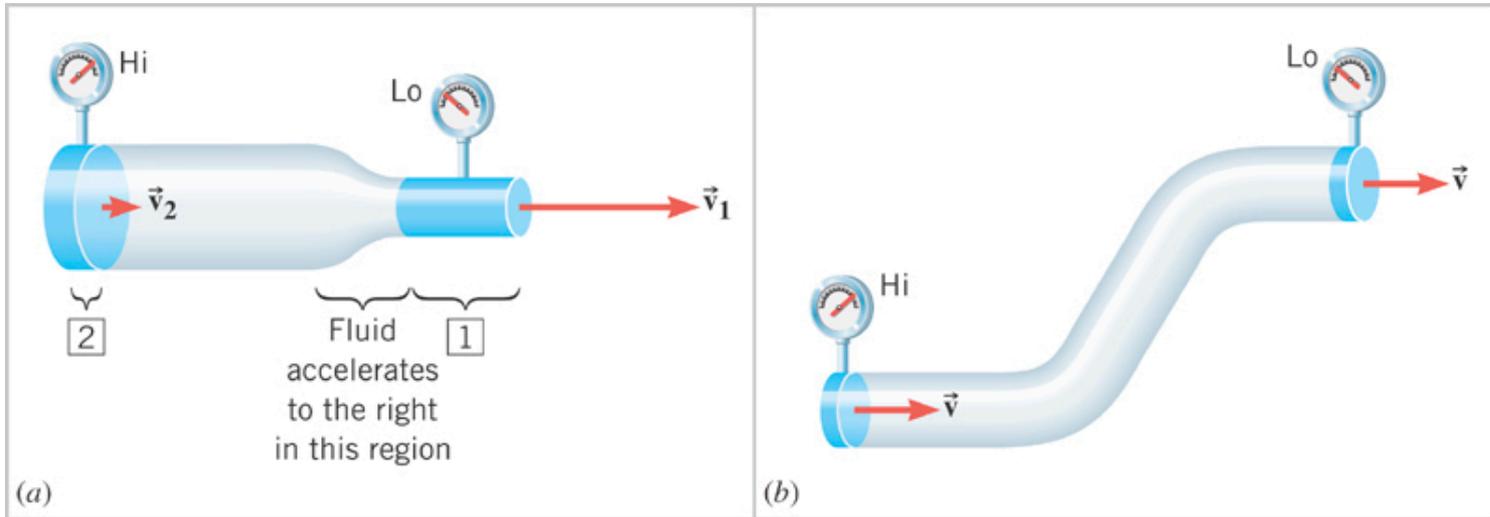
(b) $A_1 v_1 = A_2 v_2$

$$v_2 = \frac{A_1}{A_2} v_1 = (2)(0.936 \text{ m/s}) = 1.87 \text{ m/s}$$

11.9 Bernoulli's Equation

The fluid accelerates toward the lower pressure regions.

According to the pressure-depth relationship, the pressure is lower at higher levels, provided the area of the pipe does not change.

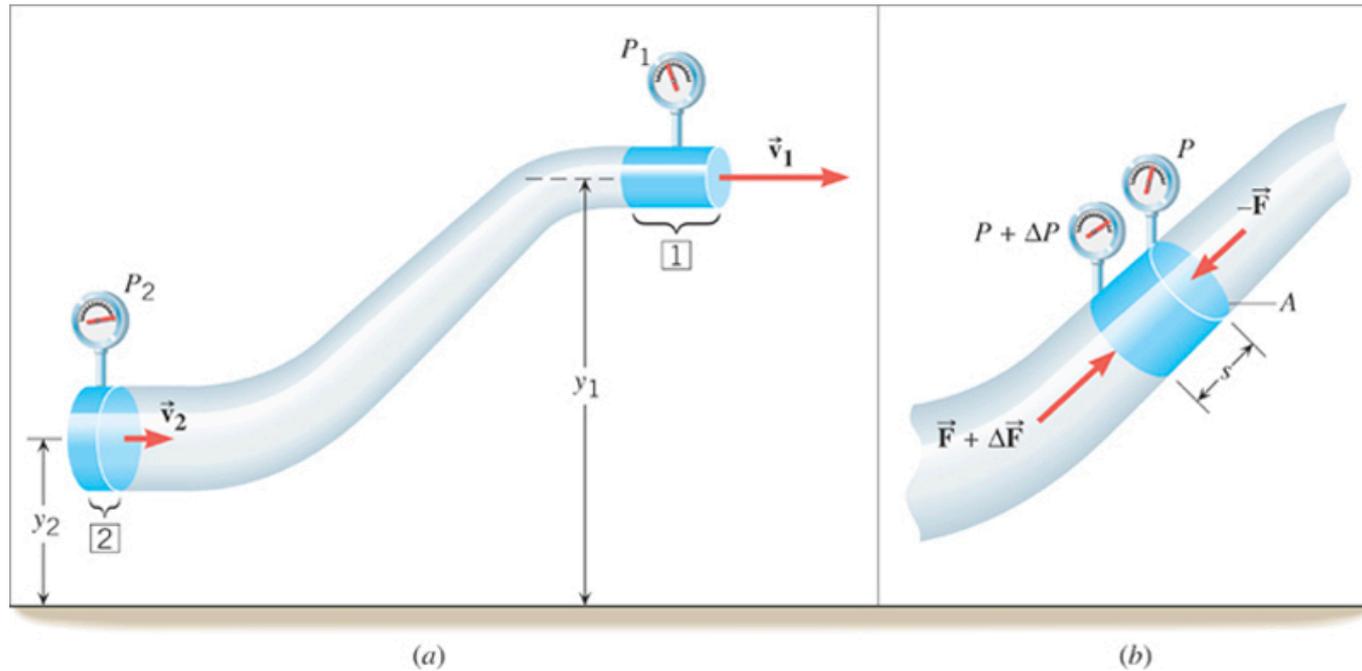


11.9 Bernoulli's Equation

$$W = (\sum F)s = (\Delta F)s = (\Delta P)As = (P_2 - P_1)V$$

Work done by pressure on the fluid (non-conservative)

$$W_{\text{nc}} = \left(\frac{1}{2}mv_1^2 + mgy_1\right) - \left(\frac{1}{2}mv_2^2 + mgy_2\right)$$



11.9 Bernoulli's Equation

$$(P_2 - P_1)V = \left(\frac{1}{2}mv_1^2 + mgy_1\right) - \left(\frac{1}{2}mv_2^2 + mgy_2\right)$$



$$(P_2 - P_1) = \left(\frac{1}{2}\rho v_1^2 + \rho g y_1\right) - \left(\frac{1}{2}\rho v_2^2 + \rho g y_2\right)$$

BERNOULLI'S EQUATION

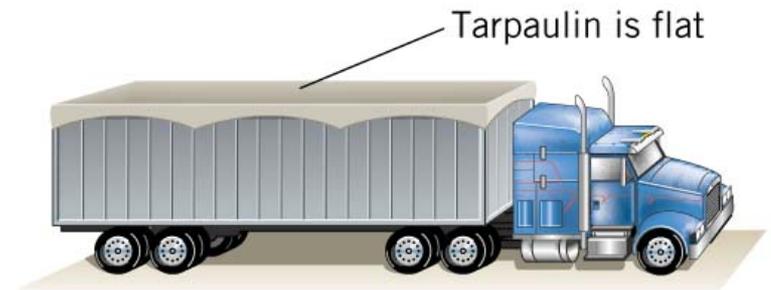
In steady flow of a nonviscous, incompressible fluid, the pressure, the fluid speed, and the elevation at two points are related by:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

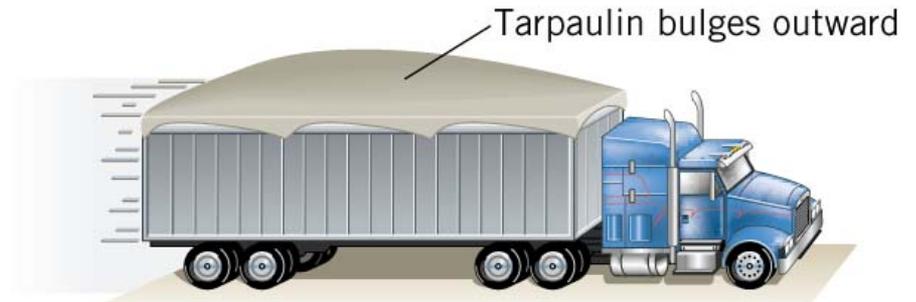
Conceptual Example 14 Tarpaulins and Bernoulli's Equation

When the truck is stationary, the tarpaulin lies flat, but it bulges outward when the truck is speeding down the highway.

Account for this behavior.

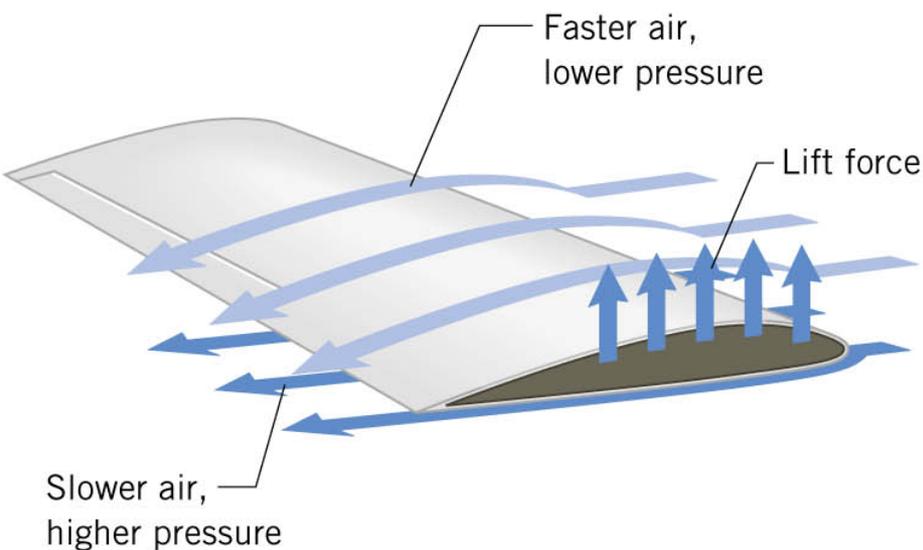


Stationary



Moving

11.10 Applications of Bernoulli's Equation

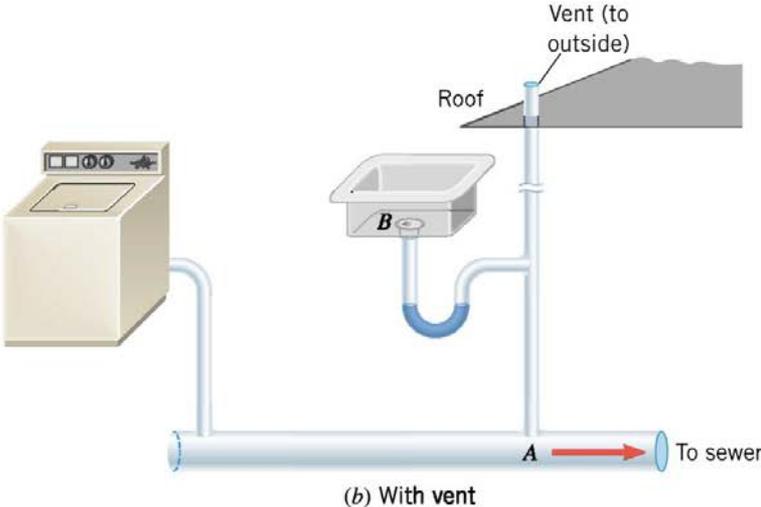
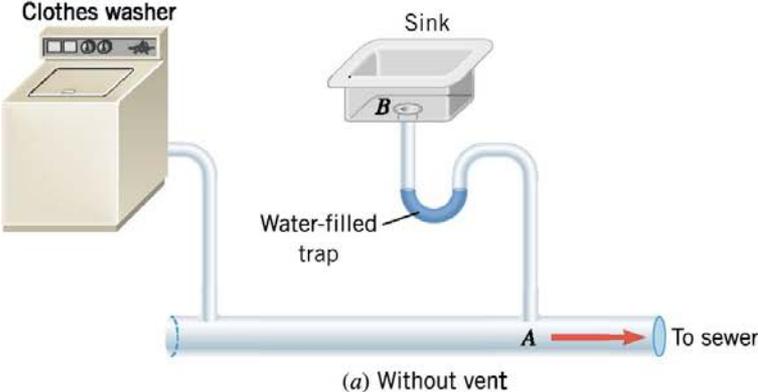


(a)

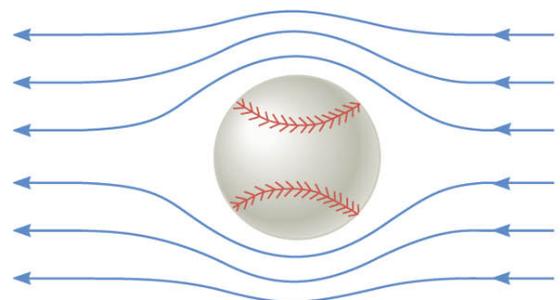


(b)

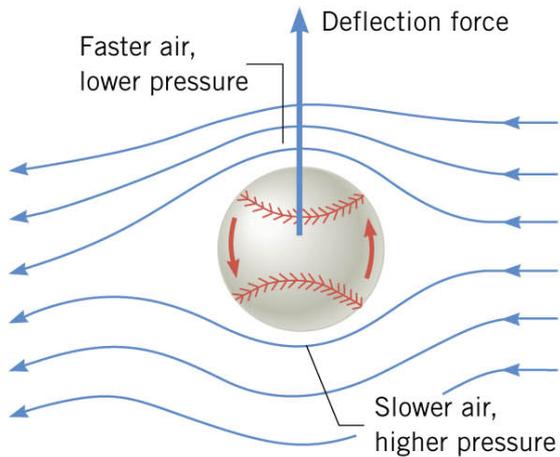
11.10 Applications of Bernoulli's Equation



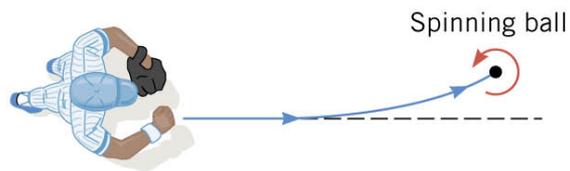
11.10 Applications of Bernoulli's Equation



(a) Without spin



(b) With spin

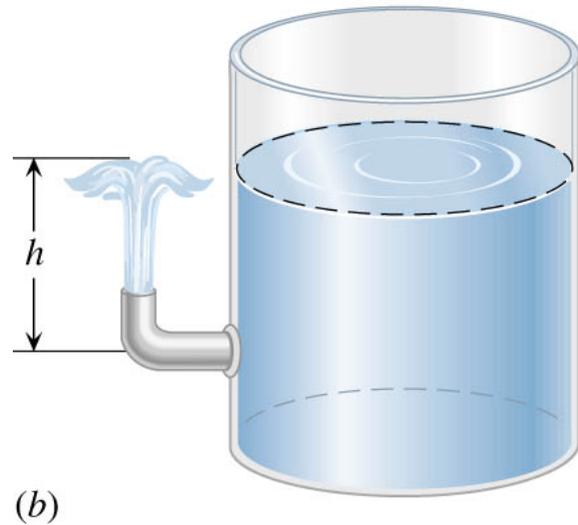
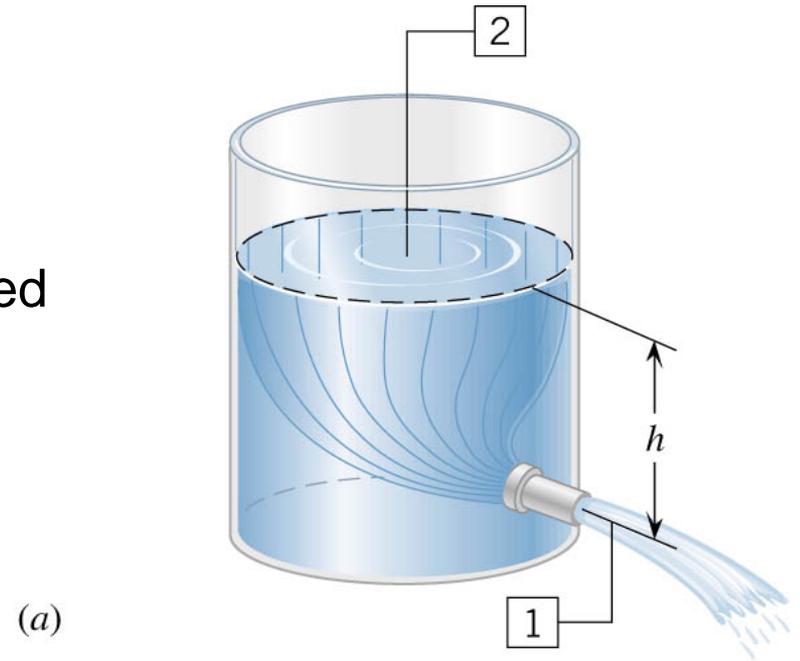


(c)

11.10 Applications of Bernoulli's Equation

Example 16 Efflux Speed

The tank is open to the atmosphere at the top. Find an expression for the speed of the liquid leaving the pipe at the bottom.



11.10 Applications of Bernoulli's Equation

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

$P_1 = P_2 = P_{atm}$

$v_2 \approx 0$

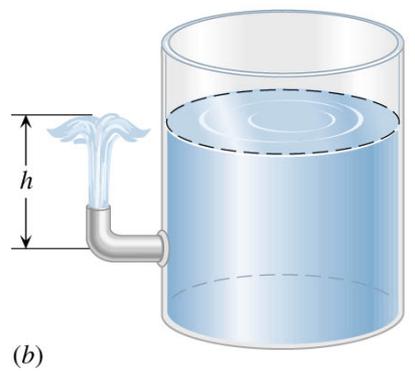
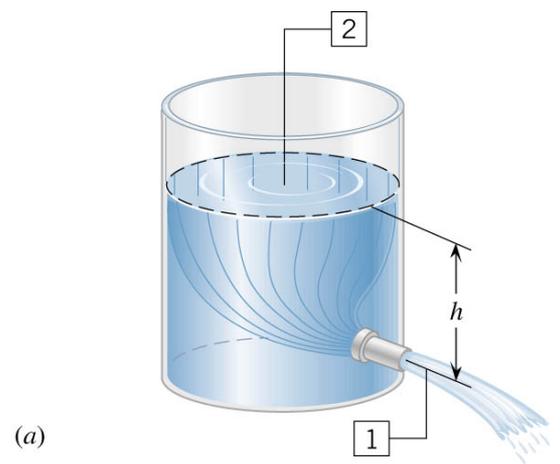
$y_2 - y_1 = h$

↓

$$\frac{1}{2} \rho v_1^2 = \rho g h$$

↓

$$v_1 = \sqrt{2gh}$$



Example 14.8

The Venturi Tube

The horizontal constricted pipe illustrated in Figure 14.19, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Determine the flow speed at point 2 of Figure 14.19a if the pressure difference $P_1 - P_2$ is known.

SOLUTION

Conceptualize Bernoulli's equation shows how the pressure of an ideal fluid decreases as its speed increases. Therefore, we should be able to calibrate a device to give us the fluid speed if we can measure pressure.

Categorize Because the problem states that the fluid is incompressible, we can categorize it as one in which we can use the equation of continuity for fluids and Bernoulli's equation.

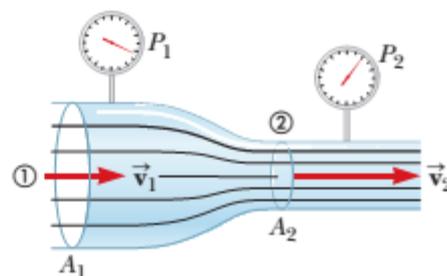


Figure 14.19 (Example 14.8) (a) Pressure P_1 is greater than pressure P_2 because $v_1 < v_2$. This device can be used to measure the speed of fluid flow. (b) A Venturi tube, located at the top of the photograph. The higher level of fluid in the middle column shows that the pressure at the top of the column, which is in the constricted region of the Venturi tube, is lower.

Analyze Apply Equation 14.8 to points 1 and 2, noting that $y_1 = y_2$ because the pipe is horizontal:

Solve the equation of continuity for v_1 :

Substitute this expression into Equation (1):

Solve for v_2 :

$$(1) \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_1 = \frac{A_2}{A_1} v_2$$

$$P_1 + \frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

Finalize From the design of the tube (areas A_1 and A_2) and measurements of the pressure difference $P_1 - P_2$, we can calculate the speed of the fluid with this equation. To see the relationship between fluid speed and pressure difference, place two empty soda cans on their sides about 2 cm apart on a table. Gently blow a stream of air horizontally between the cans and watch them roll together slowly due to a modest pressure difference between the stagnant air on their outside edges and the moving air between them. Now blow more strongly and watch the increased pressure difference move the cans together more rapidly.

Example 14.9**Torricelli's Law****AM**

An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom (Fig. 14.20). The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure P . Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance h above the hole.

SOLUTION

Conceptualize Imagine that the tank is a fire extinguisher. When the hole is opened, liquid leaves the hole with a certain speed. If the pressure P at the top of the liquid is increased, the liquid leaves with a higher speed. If the pressure P falls too low, the liquid leaves with a low speed and the extinguisher must be replaced.

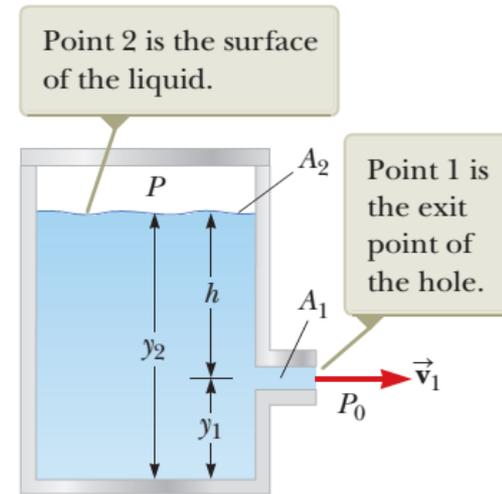


Figure 14.20 (Example 14.9) A liquid leaves a hole in a tank at speed v_1 .

Analyze Because $A_2 \gg A_1$, the liquid is approximately at rest at the top of the tank, where the pressure is P . At the hole, P_1 is equal to atmospheric pressure P_0 .

Apply Bernoulli's equation between points 1 and 2:

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

Solve for v_1 , noting that $y_2 - y_1 = h$:

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

Finalize When P is much greater than P_0 (so that the term $2gh$ can be neglected), the exit speed of the water is mainly a function of P . If the tank is open to the atmosphere, then $P = P_0$ and $v_1 = \sqrt{2gh}$. In other words, for an open tank, the speed of the liquid leaving a hole a distance h below the surface is equal to that acquired by an object falling freely through a vertical distance h . This phenomenon is known as *Torricelli's law*.

WHAT IF? What if the position of the hole in Figure 14.20 could be adjusted vertically? If the tank is open to the atmosphere and sitting on a table, what position of the hole would cause the water to land on the table at the farthest distance from the tank?

Answer Model a parcel of water exiting the hole as a projectile. From the *particle under constant acceleration* model, find the time at which the parcel strikes the table from a hole at an arbitrary position y_1 :

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

$$0 = y_1 + 0 - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y_1}{g}}$$

From the *particle under constant velocity* model, find the horizontal position of the parcel at the time it strikes the table:

$$\begin{aligned} x_f &= x_i + v_{xi}t = 0 + \sqrt{2g(y_2 - y_1)} \sqrt{\frac{2y_1}{g}} \\ &= 2\sqrt{(y_2 y_1 - y_1^2)} \end{aligned}$$

Maximize the horizontal position by taking the derivative of x_f with respect to y_1 (because y_1 , the height of the hole, is the variable that can be adjusted) and setting it equal to zero:

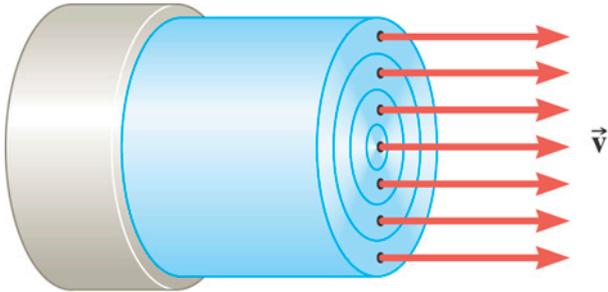
$$\frac{dx_f}{dy_1} = \frac{1}{2}(2)(y_2 y_1 - y_1^2)^{-1/2}(y_2 - 2y_1) = 0$$

Solve for y_1 :

$$y_1 = \frac{1}{2}y_2$$

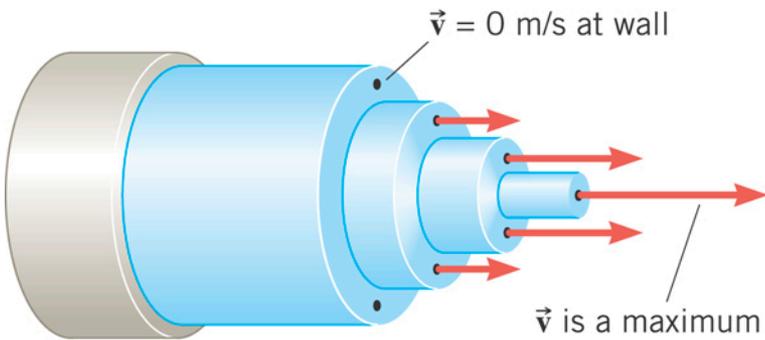
Therefore, to maximize the horizontal distance, the hole should be halfway between the bottom of the tank and the upper surface of the water. Below this location, the water is projected at a higher speed but falls for a short time interval, reducing the horizontal range. Above this point, the water is in the air for a longer time interval but is projected with a smaller horizontal speed.

11.11 Viscous Flow (not on the final exam)



(a)

Flow of an ideal fluid.



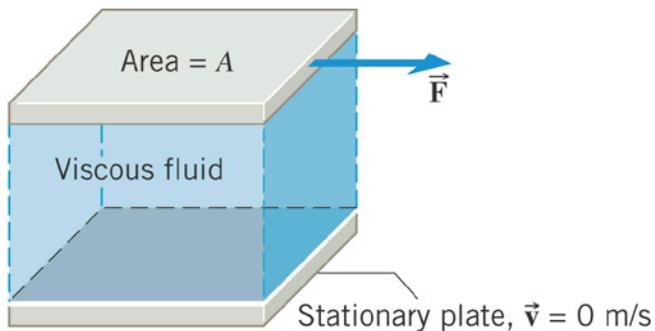
(b)

Flow of a viscous fluid.

11.11 Viscous Flow

FORCE NEEDED TO MOVE A LAYER OF VISCOUS FLUID WITH CONSTANT VELOCITY

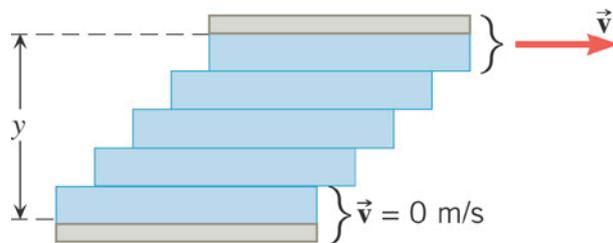
The magnitude of the tangential force required to move a fluid layer at a constant speed is given by:



(a)

$$F = \frac{\eta A v}{y}$$

coefficient
of viscosity



(b)

SI Unit of Viscosity: Pa·s

Common Unit of Viscosity: poise (P)

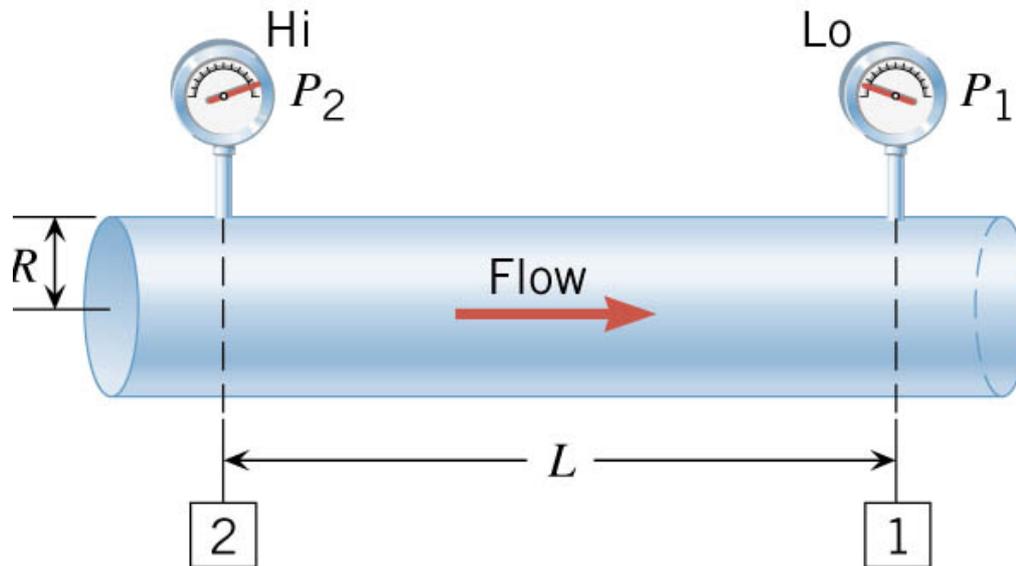
1 poise (P) = 0.1 Pa·s

11.11 Viscous Flow

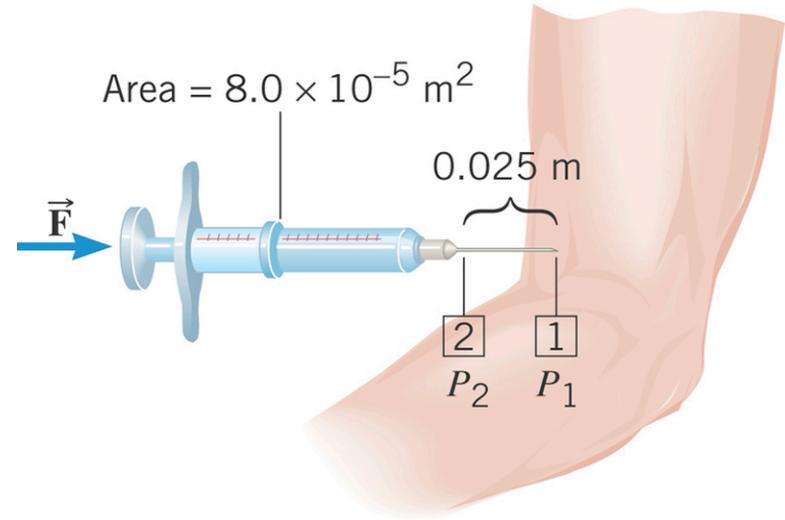
POISEUILLE'S LAW

The volume flow rate is given by:

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta L}$$



11.11 Viscous Flow

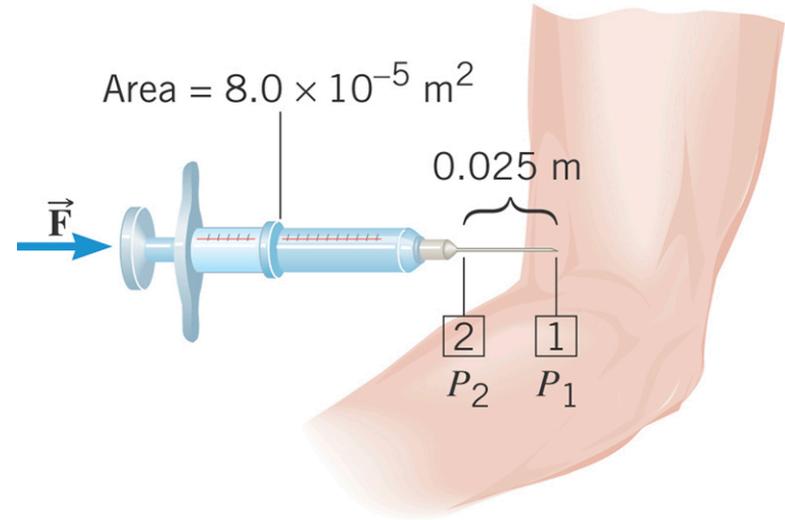


Example 17 Giving and Injection

A syringe is filled with a solution whose viscosity is $1.5 \times 10^{-3} \text{ Pa}\cdot\text{s}$. The internal radius of the needle is $4.0 \times 10^{-4} \text{ m}$.

The gauge pressure in the vein is 1900 Pa . What force must be applied to the plunger, so that $1.0 \times 10^{-6} \text{ m}^3$ of fluid can be injected in 3.0 s ?

11.11 Viscous Flow



$$P_2 - P_1 = \frac{8\eta LQ}{\pi R^4}$$

$$= \frac{8(1.5 \times 10^{-3} \text{ Pa} \cdot \text{s})(0.025 \text{ m})(1.0 \times 10^{-6} \text{ m}^3 / 3.0 \text{ s})}{\pi(4.0 \times 10^{-4} \text{ m})^4}$$

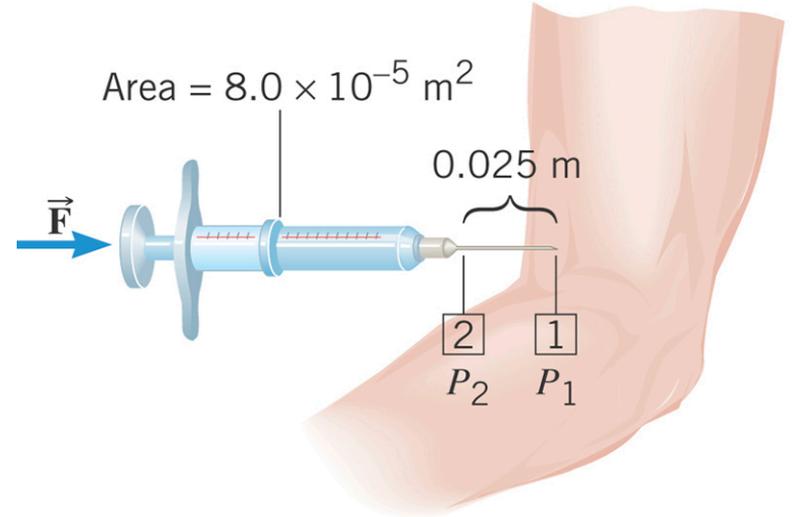
$$= 1200 \text{ Pa}$$

11.11 Viscous Flow

$$P_1 = 1900 \text{ Pa}$$
$$P_2 - P_1 = 1200 \text{ Pa}$$



$$P_2 = 3100 \text{ Pa}$$



$$F = P_2 A = (3100 \text{ Pa})(8.0 \times 10^{-5} \text{ m}^2) = 0.25 \text{ N}$$

Summary

- How to describe continuum of matter → field variables
 - Density (instead of mass)
 - Pressure or stress (instead of force)
 - Scalar quantity, exert in all direction on the container
 - particle velocity (vs current flux)
- Fluid pressure due to gravity
 - Archimedes principle
- Ideal fluid (incompressible, non-viscous), steady/laminar flow vs turbulent flow
- Conservation of mass (mass flow rate, continuity equation)
- Work-energy theorem for fluid (Bernouille's equation)